9

DESIGN OF COMPRESSION MEMBERS

INTRODUCTION

A structural member which is subjected to compressive forces along its axis is called a *compression member*. Thus, compression members are subjected to loads that tend to decrease their lengths. Except in pin-jointed trusses, such members (in any plane or space structure), under external loads, experience bending moments and shear forces. If the net end moments are zero, the compression member is required to resist load acting concentric to the original longitudinal axis of the member and is termed *axially loaded column,* or simply *column.* If the net end moments are not zero, the member will be subjected to an axial load and bending moments along its length. Such members are called *beam-columns* and are treated in Chapter 13.

Let us consider an example of an axially loaded column shown in Fig. 9.1. For this column, the axial load P, to be resisted by it, is the sum of the beam shears $V_1 + V_2$. For the column shown in Fig. 9.1, the net end moment is assumed to be zero; this is true if the end moments and shears developed by the two beams are equal. Such situations arise in many interior columns of buildings having equal column spacing. Where the beam is not connected rigidly to the column, the beams will not develop significant end moments and in such situations also the column has to resist only the difference in end shears. In several interior columns, the net moment will be small and the member is designed as an axially loaded column.

Multi-storey steel building under construction in Maryland, USA

Close-up view of the column of the above building (Note the composite slab)

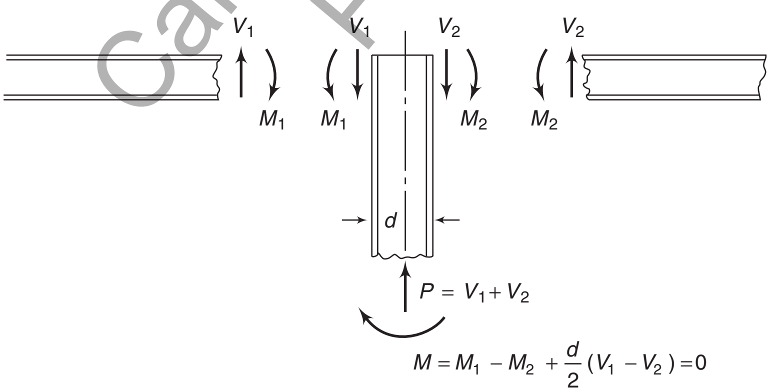


FIG. 9.1 Axially loaded column

Different terms are used for a compression member depending upon its position in a structure. The vertical compression members in a building supporting floors or girders are normally called as columns (referred sometimes as *stanchions* in UK). They are subjected to heavy loads. Sometimes vertical compression members are called *posts*. The compression members used in roof trusses and bracings are called *struts*. They may be vertical or inclined and normally have small lengths. The top chord members of a roof truss are called the *principal rafter.* The principal compression member in a crane is called the *boom.* Short compression members at the junction of columns and roof trusses or beams are called *knee braces.* Some of these compression members are shown in Fig. 9.2.

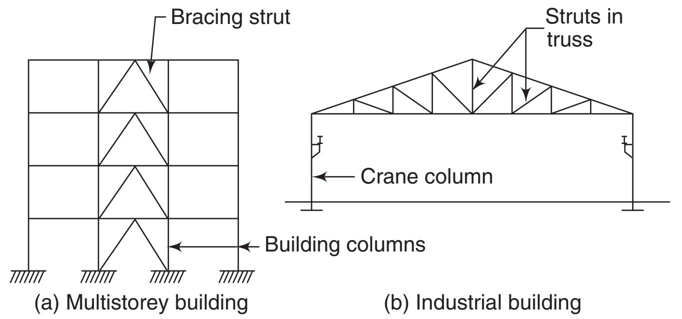


FIG. 9.2 Types of compression members

It is well known from basic mechanics of materials that only very short columns can be loaded up to their yield stress. For long columns (see Section 9.4.1 also), *buckling* (deformation in the direction normal to the load axis) occurs prior to developing the full material strength of the member. Hence, a sound knowledge of stability theory is necessary for designing compression members in structural steel.

Since compression members have to resist buckling, they tend to be stocky and 'square' and circular tubes are found to be the ideal sections, since their radius of gyration is same in the two axes (see also Section 9.8). This situation is in contrast to the slender and more compact tension members and deep beam sections. Unlike the member subjected to tension, a compression member is designed on the assumption that its gross cross-sectional area will be effective in resisting the applied loads. Bolts may be used to connect columns to adjacent members. As the load is applied, the member will contract. It is assumed that the action of bolts is such that they will replace the material removed for holes. Thus, the bolt holes are often ignored in the design. Since compression members comprise of thin plates, they also experience local buckling, as discussed in Chapter 8.

9.1 CONSTRUCTION DETAILS

Columns in buildings are connected to other members and foundation through the following:

1. Beam-to-column connections
2. Column cap connections
3. Column base plates

Column splices are used to increase the height of columns or to connect different column cross sections.

Typical beam-to-column connections are discussed in Chapters 5, and 14 (see Figs 5.1, 5.2, 14.27, and 14.28) and column cap connections are shown in Fig. 9.3. Splices in compression members are discussed in Sections 5.9 and 6.12.2 (see Figs 5.41, 5.42, and 6.56). Column base plates which are used to connect columns with concrete foundations are discussed in Chapter 15.

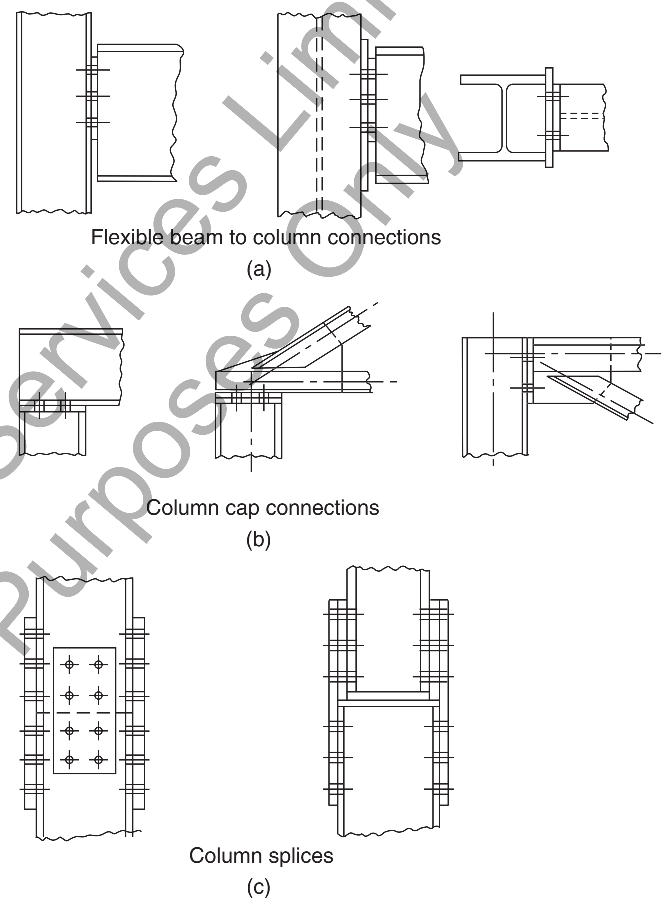


FIG. 9.3 Typical column connections

9.1.1 Loads on Compression Members

As discussed previously, axial loading on columns in buildings is due to loads from roofs, floors, and walls transmitted to the column through beams and also due to its own self-weight, as shown in Fig. 9.4(a). Floor beam reactions are often eccentric to the column axis [see Fig. 9.4(a)] and if either the beam arrangement or the loading is asymmetrical will result in moments to be transmitted to the column. Wind loads in multi-storey buildings are usually applied at the respective floor levels and are assumed to be resisted by the bracings and hence in braced buildings do not cause large moments. However, in unbraced rigid framed buildings, the moments due to wind loads should also be taken into account in the design of columns [see Fig. 9.4(b)].

specifications usually have rules to prevent such failures (see Section 9.15).

9.3 CLASSIFICATION OF CROSS SECTION

If individual plate elements which make up the cross section of a compression member, (for example, the web and two flanges in the case of an I-section), are thin, local buckling of the type shown in Fig. 9.5 may occur. For columns, it is frequently possible to eliminate this problem by limiting the proportions of component plates, such that local buckling will not influence the strength of the cross sections.

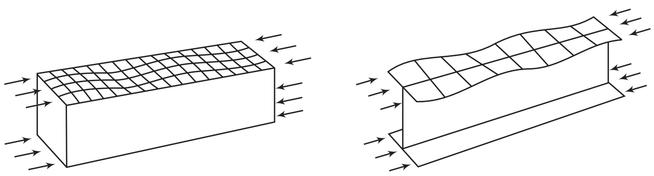


FIG. 9.5 Local buckling in box- or I-section columns (deformation of a single flange only is shown for clarity)

The same classification that was set out for beams, in Section 8.13, is used for compression members. That is, to prevent local buckling, the width-to-thickness ratio of the different elements (web, flange, etc.,) of the compression member should be limited. Upper limits for the limiting width-to-thickness ratios for elements subjected to compression are given in Table 9.1. These limits are applicable to semi-compact sections. When more slender plates, having width-to-thickness ratios higher than the limits given in Table 9.1 are to be used, the strength of the section should be suitably reduced. However, it is a good practice to use plastic or compact sections for compression members, since they provide more stiffness than semi-compact or slender sections.

TABLE 9.1 Limiting width-to-thickness ratios for axial compression elements

Element	Ratio	Upper limit for semi-compact section
Outstand element of compression flange (I, H, or C)		
(a) Rolled section	b/t_f	15.7ε
(b) Welded section	b/t_f	13.6ε
Internal element of compression flange (box)	b/t_f	42ε
Web of an I-H or box-section	d/t_w	$126/(1+2r_2) \geq 42\varepsilon$
Single angle or double angle with the components separated (all three criteria should be satisfied)	b/t d/t $(b+d)/t$	15.7ε 15.7ε 25ε
Circular hollow section	D/t	$88\varepsilon^2$
Hot rolled RHS—Flange	b/t	42ε
Web	d/t	$126/(1+2r_2) \geq 42\varepsilon$
Cold formed RHS—Flange	b/t	36.7ε
Web	d/t	$110/(1+2r_2) \geq 37\varepsilon$

$\varepsilon = (250/f_y)^{0.5}$
For HR RHS: $b = B - 3t$; $d = D - 3t$
For CF RHS: $b = B - 5t$; $d = D - 5t$

9.4 BEHAVIOUR OF COMPRESSION MEMBERS

Before discussing the behaviour of compression members, it may be useful to know about the classification of compression members based on their length.

9.4.1 Long, Short, and Intermediate Compression Members

Compression members are sometimes classified as being long, short, or intermediate. A brief discussion about this classification is as follows:

Short compression members For very short compression members the failure stress will equal the yield stress and no buckling will occur. Note that for a compression member to fall into this classification, it has to be so short (for an initially straight column $L \leq 88.85r$, for $f_y = 250$ MPa) that it will not have any practical application.

Long compression members For these compression members, the Euler formula [see Section 9.5 and Eqn (9.9)] predicts the strength very well, where the axial buckling stress remains below proportional limit. Such compression members will buckle elastically.

Intermediate length compression members For intermediate length compression members, some fibres would have yielded and some fibres will still be elastic. These compression members will fail both by yielding and buckling and their behaviour is said to be 'inelastic'. For the Euler formula Eqn (9.9) to be applicable to these compression members, it should be modified according to the reduced modulus concept or the tangent modulus concept (as is done in AISC code formula) to account for the presence of residual stresses. Now let us discuss the behaviour of short and slender columns.

9.4.2 Short Compression Members

Consider an axially compressed member of short length which is initially straight and made of material having the ideal rigid–plastic stress–strain relationship as shown in Fig. 9.6. At low values of external load P, there will be no visible deformation—neither lateral nor axial. Since P is applied at the centroid of the section, apart from possible localized effects at the ends of the member, all parts of the member will experience the same value

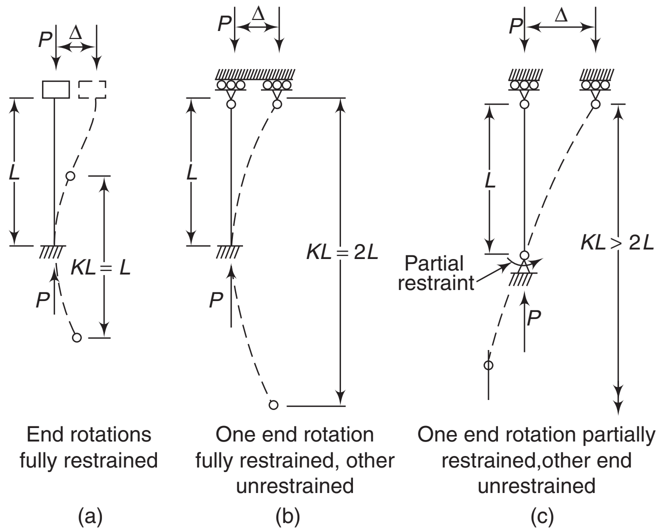


FIG. 9.26 Effective length *KL* when there is joint translation at the ends

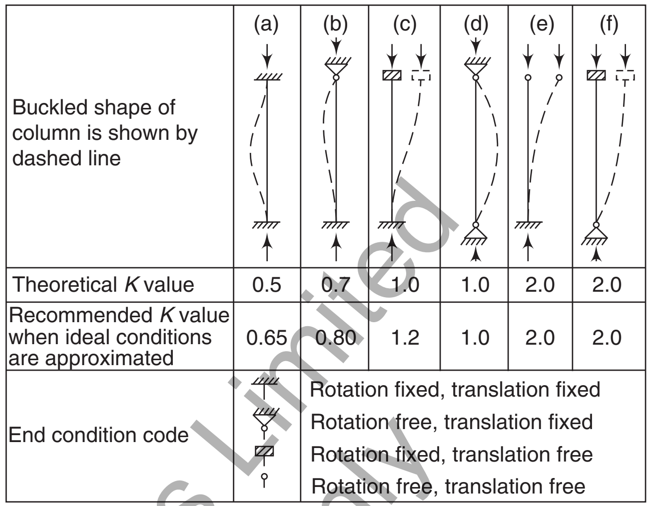


FIG. 9.27 Effective-length factors *K* for centrally loaded columns with various end conditions

TABLE 9.6 Effective length factors for columns in various codes of practice

End condition (see Fig. 9.27)	(a)	(b)	(c)	(d)	(e)	(f)
IS 800-2007	0.65	0.80	1.20	1.0	2.0	2.0
BS: 5950-Part1: 2000	0.70	0.85	1.20	1.0	2.0	—
AS 4100-1998	0.70	0.85	1.20	1.0	2.2	2.2
CAN/CSA-S16.1-2001	0.65	0.80	1.20	1.0	2.0	2.0
ANSI/AISC 360-2005	0.65	0.80	1.20	1.0	2.1	2.0
AIJ, 1970	0.65	0.80	1.20	1.0	2.1	—

where K = 1.0 for columns with both ends pinned,

K = 0.5 for columns with both ends fixed,

K = 0.707 for columns with one end fixed and the other end pinned,

K = 2.0 for columns with one end fixed and the other end free,

K ≤ 1.0 for columns partially restrained at each end, and

K ≥ 2.0 for columns with one end unstrained and the other end rotation partially restrained.

Thus, the effective length is important in design calculations because the buckling load is inversely proportional to the square of the effective length.

Approximate values for effective lengths, which can be used in design, are given in IS 800 and are shown in Fig. 9.27 along with the theoretical *K* values [as given by Eqn (9.43)]. Note that the values given in the code are slightly more than the theoretical values given in (a), (b), and (c) of Fig. 9.27. It is because, fully rigid end restraints are difficult to achieve in practice; partial end restraints are much more common in practice. (The buckling lengths given in Fig. 9.27 are not to be applied to angles, channels, or T-sections, as per BS: 5950-Part 1.) For example, at the base, shown fixed for conditions (a), (b), (c), and (e) in Fig. 9.27, full fixity can be assumed only when the column is anchored securely to a footing, for which rotation is negligible. (Individual footings placed on compressible soils, will rotate due to any slight moment in the column.) Similarly, restraint conditions, (a), (c), and (f) at the top can be achieved only when the top of the column is framed integrally to a girder, which may be many times stiffer than the column. Condition (c), as shown in Fig. 9.27 is applicable to columns supporting heavy loads at the top (e.g., columns supporting storage tanks). Various effective length factors suggested by different codes of practice are given in Table 9.6.

9.9.2 Intermediate Restraints and Effective Lengths in Different Planes

In the previous sections, it was assumed that the compression member was supported only at its ends as shown in Fig. 9.27. If the member has an additional lateral support (bracing) which prevents it from deflecting at its centre, as shown in Fig. 9.28, the elastic buckling load is increased by a factor of four (i.e., the value of P_{cr} will be $4\pi^2EI/L^2$).

This restraint need not be completely rigid, but may be elastic, provided its stiffness exceeds a certain minimum value. This value of minimum stiffness k_b has been derived by Salmon & Johnson (1996) as

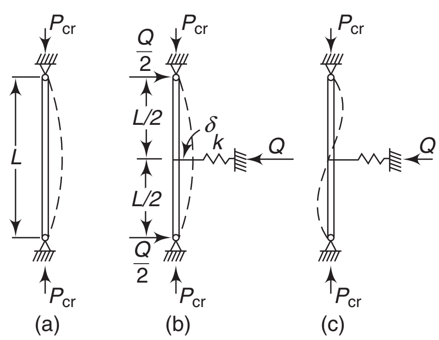


FIG. 9.28 Compression member with an elastic intermediate restraint

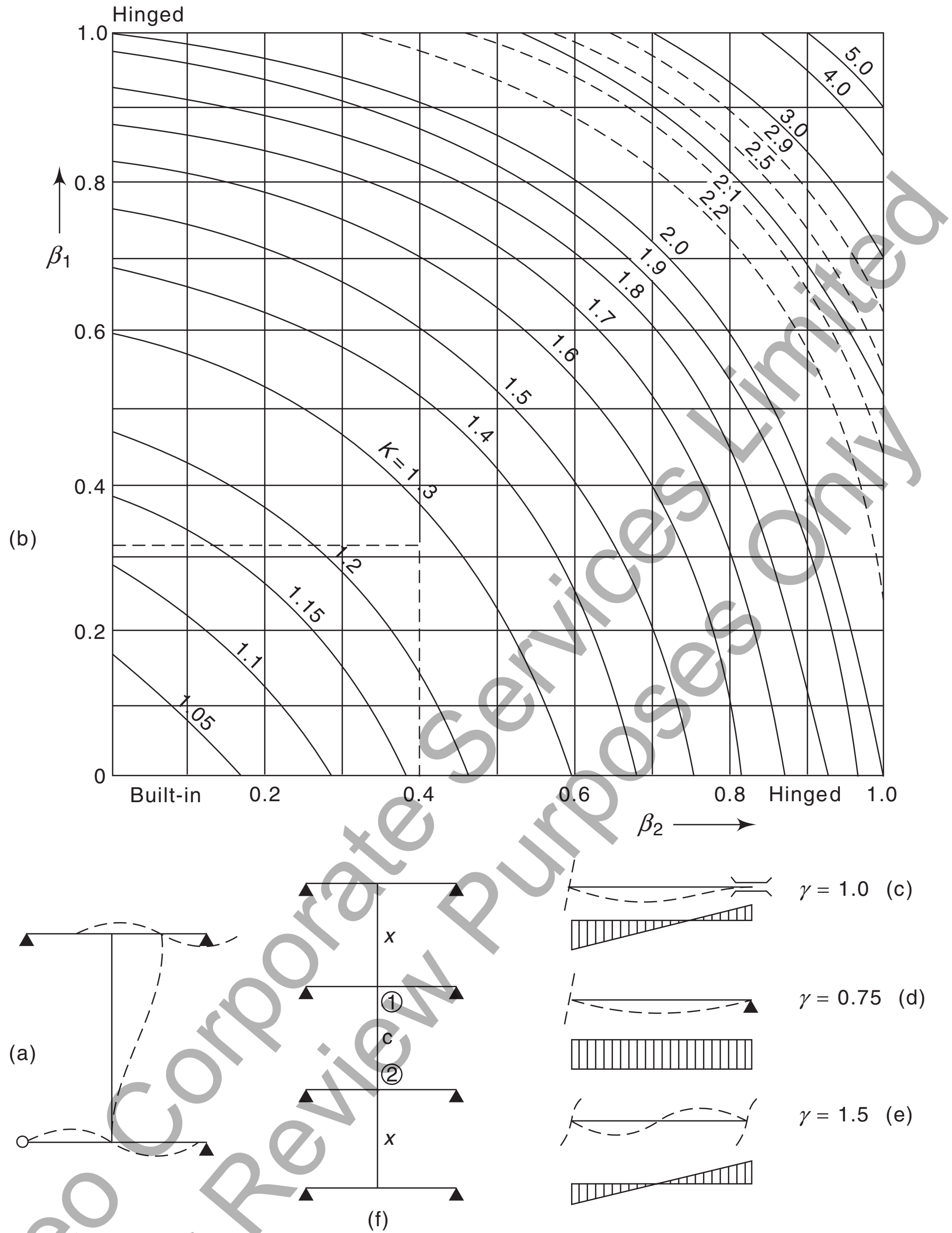


FIG. 9.34 Wood's curves for the determination of effective length for unbraced frame

for the columns. Such correction curves have been developed by Chen and Chow (1969). More discussions about these curves, including worked examples are provided by Johnston (1976).

In the models for the braced frame in Fig. 9.30(a) and for the unbraced frame in Fig. 9.30(b), it was assumed that the axial forces are constant for all columns and axial forces in all beams are negligible. This condition can be relaxed for braced frames, using the procedure developed by Bridge and Fraser (1987).

The following equation was proposed by LeMessurier (1972) for calculating the effective length factor of member i in a fram.

$$K_i = \sqrt{\left\{(\pi^2 I_i / P_i)[(\sum P + \sum C_L P)/\sum(\beta I)]\right\}} \qquad (9.50a)$$

In the preceding equation, I_i is the moment of inertia of member i (whose effective length factor K_i is required), P_i is the axial force in member i (calculated based on a first-order analysis of the frame), ΣP is the total gravity load on the storey, and β is a factor to account for the effect of various restraining conditions at the ends of the column. The value of β is given by

$$\beta = \frac{[6(G_A + G_B) + 36]}{[2(G_A + G_B) + G_A G_B + 3]} \qquad (9.50b)$$

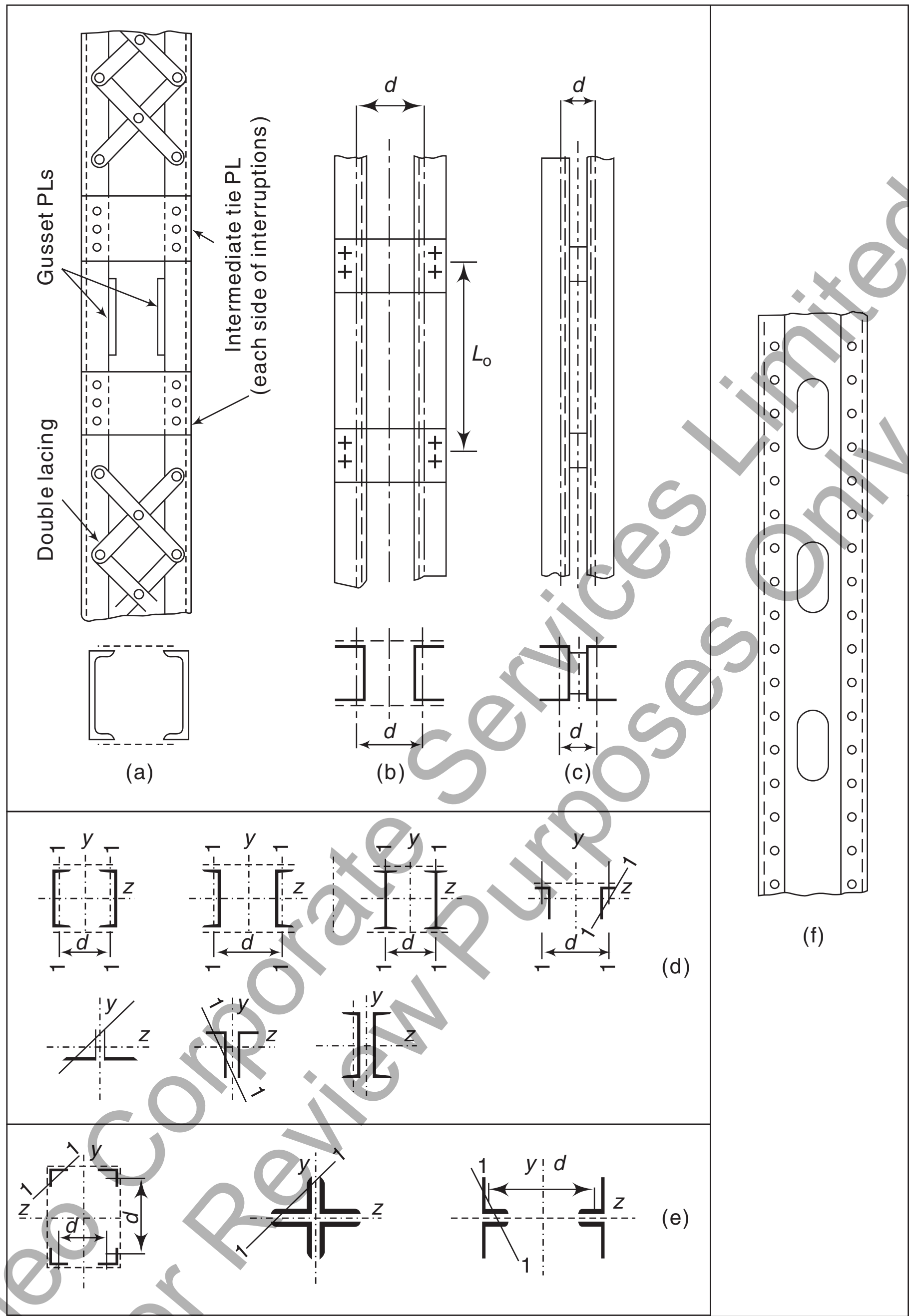


FIG. 9.46 Built-up columns

load for a built-up column is less than that of a comparable solid column because the effect of shear on deflections is much greater for the former (Galambos 1998). The shear in column may be due to the following:

1. Lateral loads from wind, earthquake, gravity, or other causes
2. The slope of column with respect to the line of thrust due both to unintentional initial curvature and the increased curvature during buckling
3. The end eccentricity of the load due to either end connections or fabrication imperfections

The slope effect is most important for slender columns and the eccentricity effect for short columns. Lin et al. (1970) suggested a shear flexibility factor μ, using which the equivalent slenderness λ_e can be computed and using curve *c* of Fig. 9.23, the design stress can be computed (for more details see Lin et al. 1970; Galambos 1998).

For columns with batten plates, the strut may be designed as a single integral member with a slenderness given by (Bleich 1952)

$$\lambda_e = \sqrt{\left[\lambda_m^2 + \lambda_e^2\left(\pi^2/12\right)\right]} \tag{9.62}$$

With the limitations $\lambda_e > 50$ and $\lambda_e < 1.4\ \lambda_c$ where $\lambda_m = L/r_{\min}$ = strut slenderness, $\lambda_c = L_o/r_o$ = local chord slenderness between one batten plate and the next, and L_o is the centre-to-centre distance of batten plate.

This equation was derived strictly for hinged-end columns. Bleich estimated that the buckling strength of a steel column having L/r of 110 is reduced by about 10% when λ_c = 40 and by greater amounts for larger values of λ_c. Aslani and Goel (1991) proposed the following equation based on their theoretical and experimental investigations, which is applicable to all built-up columns with general end conditions

$$\lambda_e = \sqrt{\left[\lambda_m^2 + 0.82\left(\frac{\alpha^2}{1+\alpha^2}\right)\lambda_c^2\right]} \tag{9.62a}$$

Where $\alpha = d/(2r_{ib})$ and r_{ib} is the radius of gyration of individual components relative to its centroidal axis parallel to member axis of buckling and d is the distance between centroids of individual components. Equation 9.62(a) was adopted in the AISC 360 code (from 1993 to 2005) for intermediate connectors that are welded or having pre-tensioned bolts. However, the AISC 360-2010 code specifies the following equations based on the statistical evaluations by Sato and Uang (2007).

1. When intermediate connectors are bolted snug tight

$$\lambda_e = \sqrt{\left[\lambda_m^2 + \lambda_c^2\right]} \tag{9.62b}$$

2. When intermediate connectors are welded or connected by means of pre-tensioned bolts
   (a) When $\lambda_c \le 40$

$$\lambda_e = \lambda_m \tag{9.62c}$$

   (b) When $\lambda_c > 40$

$$\lambda_e = \sqrt{\left[\lambda_m^2 + \left(K_i\lambda_c\right)^2\right]} \tag{9.62d}$$

where K_i = 0.5 for angles back-to-back
= 0.75 for channels back-to-back
= 0.86 for all other cases

The most commonly adopted lacing systems are shown in Fig. 9.47. The simplest form of lacing consists of single bars connecting the two components [Fig. 9.47(b)]. The double lacing [Fig. 9.47(d)] is sometimes considered preferable,

Built-up column in a portal frame, Mumbai (Note the stubs in the column, which carry a small crane)

(a) General view under construction (b) View of the lacings

4 ×15 m span continuous *lattice portal* frame for Kirloskar small engines division, Rajkot (*Note* how the lacings are welded inside the angles and the staggering on perpendicular faces)

Courtesy: Er Jayant Lakhlani

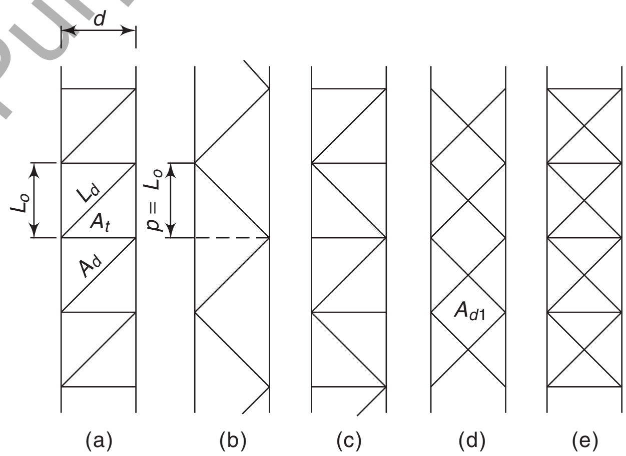


FIG. 9.47 Commonly adopted lacing systems [(c) and (e) are not recommended]

Struts with *perforated* cover plates used in the Carquinez cantilever truss bridge (East bound I-80), California, USA

Photo: Sreemathi Subramanian

although a well-designed single lacing is equally effective. In single or double lacing systems, cross members perpendicular to the longitudinal axis of the strut should not be used [see Figs 9.47(c) and (e)]. The 'accordion' like action of the lacing system without cross members permits the lateral-expansion of the column. The introduction of cross members prevents the lateral expansion and thus forces the lacing bar to share the axial load on the strut. Note that the *lacing bars* and *batten plates* are not designed as load carrying elements. Their function is primarily to hold the main component members of the built-up column in their relative position and equalize the stress distribution in them. At the ends and at intermediate points where it is necessary to interrupt the lacing (for example, to admit gusset plates), the open sides are connected with *tie plates* (also called batten plates or stay plates). Tie plates are also provided at the top and bottom of the column (see Fig. 9.48).

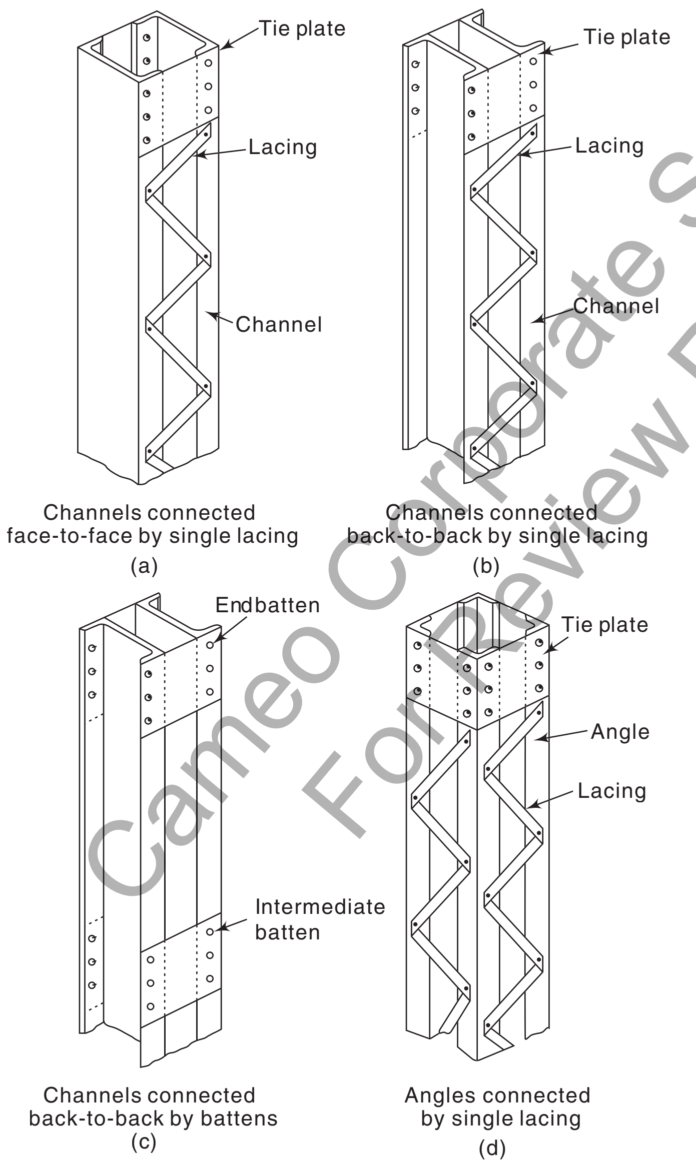


FIG. 9.48 Isometric view of built-up columns

Saboo and Rai (2004), based on their experimental work, found that the column with the batten spacing as per IS 800 may not perform satisfactorily when subjected to earthquakes. They suggest the spacing of battens has to be reduced to half in the plastic hinge zones.

It should be noted that the battened columns have the least resistance to shear compared to columns with lacings and perforated plates, and may experience an appreciable reduction in strength. Hence, they are not generally used in the United States. Columns with perforated plates require no special considerations for shear effects. These cover plates contain perforations spaced axially, which afford access for welding or painting. After the advent of automatic cutting machines, the production of such perforated plates have become simpler. Hence, they are used extensively in USA. They result in reduction of fabrication and maintenance cost and offer superior stiffness and straightness. At present, they are not used in India and interested readers may refer to Salmon and Johnson (1996) for the design of built-up columns with perforated plates.

The effective slenderness of laced struts of the types shown in Figs 9.47(c), (d), and (e) (with two chords connected by lacings) may be obtained by using the following equation (Ballio & Mazzolani 1983)

$$\lambda_{eq} = \sqrt{\left[\lambda^2 + \pi^2 (A/A_d) L_d^3/(L_o d^2)\right]} \qquad (9.63)$$

where λ is the strut slenderness $= L/r_{\min}$, A is the overall cross-sectional area $= 2A_1$, A_1 is the area of the individual chord, A_d is the cross-sectional area of the diagonal lacing $\{= 2A_{d1}$ for Figs 9.47(d) and (e)$\}$, L_d is the length of the diagonal lacing, d is the distance between the centroid of the chords, and L_o is the chord length between the two successive joints.

Equation (9.63) is also applicable to the laced strut type shown in Fig. 9.47(b), provided L_o is replaced by p.

If two I-sections of area A, moments of inertia (I_z and I_y), and radii of gyration (r_z and r_y) about the axes Z and Y respectively, and are kept side by side at a centre-to-centre distance of x as shown in Fig. 9.48(a), their combined moment of inertia will be

$I_{zz} = 2I_z$ Hence $r_{zz} = r_z$

$$I_{yy} = 2\left[I_y + A\left(\frac{x}{2}\right)^2\right]$$

Equating I_{zz} and I_{yy} for maximum efficiency, we get

$$2I_z = 2[I_y + A\left(\frac{x}{2}\right)^2]$$

or

$$\left(\frac{x}{2}\right)^2 = \frac{I_z - I_y}{A} = r_{zz}^2 - r_{yy}^2$$

Hence $x \approx 2r_{zz}$ as r_{yy} is much smaller than r_{zz}.

This formula can be used to find the optimum spacing of two I-sections in built-up columns.

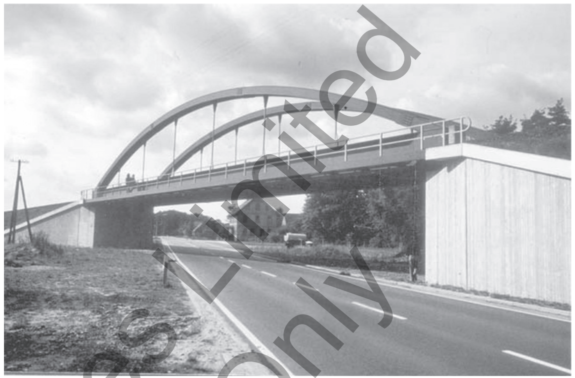
Arch bridge connecting USA and Canada near Niagara Falls, USA (note the bracings between the two arches)

Fig. 9.51(c). Thus, a pair of arches at either end of a structure may be stabilized through the use of diagonal elements. Interior arches may be stabilized by connecting them to the end arches by transverse members. Twin arch ribs with a lateral bracing system often occur in arch bridges.

The second major problem with respect to the behaviour of arches in the lateral direction is that of lateral buckling. Since internal forces are often fairly low, the elements of a steel arch can be fairly slender. Due to this, an out-of-plane buckling, as shown in Fig. 9.51(b) can occur. This problem may be solved either by increasing the stiffness of the arch in the lateral direction (by increasing the lateral dimension) or by bracing the arch periodically along its length with transverse members. Thus, the same system that is used to stabilize the arch from overturning laterally [Fig. 9.51(c)] also provides lateral bracing for arch members and prevents lateral buckling. If non-uniform loading or point loads are applied to arches, bending is developed in arch members in addition to axial forces. The higher the bending, the larger must be the member used.

For three-hinged arches, the internal shears, moments, and reactions can be found out by the direct application of the basic equations of statics. The two-hinged arch and the fixed-ended arch are more complicated to analyse and require statically indeterminate analysis. They could be analysed using plane frame computer programs. The three-hinged arch is least affected by support settlements (due to sinking, earthquakes, etc.) and temperature expansion or contraction, whereas the fixed-ended arch is the most affected by these conditions. However, the fixed-ended arch is the most preferable for deflection control, followed by the two-hinged arch. The three-hinged arch is sensitive to deflections, since the presence of the hinges reduce the overall stiffness of the arch. Thus, the two-hinged arch is frequently used because it combines some of the advantages of the other two types of arches while not comparably sharing their disadvantages (Schodek 2001).

The analysis and design of arches may be broadly classified according to their response to load and their in-plane mode of failure. These characteristics depend on the type of arch, such as the following (Galambos 1998):



Arch bridge in Germany

1. Slender arches, generally of solid web with rolled or built-up sections, subjected primarily to axial forces in the arch rib (similar to an axially loaded column)
2. Slender arches subjected to significant bending and deformation due to asymmetric or point loading (similar to beam-column problems as discussed in Chapter 13)
3. Stocky arches, with truss-like configuration, which are used when heavy bending occurs. In these arches, the failure will be primarily due to chord or flange failure arising from axial load and excessive bending (similar to any truss subjected to axial force and bending moment)
4. Arches composed of arch ribs and deck stiffening girders

The in-plane and out-of-plane (linear and non-linear) buckling behaviour is outside the scope of this book and interested readers may refer to Galambos (1998). Design procedures for arches based on ultimate inelastic strength studies, and using interaction type formula similar to beam-column formulas have been proposed by several authors. (e.g., Sakimoto & Komatsu 1983) and a review of these methods is provided by Galambos (1998).

As discussed earlier, most arches in practice are braced against lateral movement either continuously or at regularly spaced intervals. When pairs of arch ribs are braced closely with stiff transverse bars, the critical load per one arch rib of a set of braced arches can be increased up to 250% of that for the identical isolated single arch, so far as elastic buckling is concerned. For actual bridge arches, this increase may be limited by yielding of the material. The strength of braced arches in the plane of bracing is similar to the buckling strength of columns with lacing bars and battens (Timoshenko & Gere 1961). Simple approximate methods for determining strength of braced on unbraced steel arches which fail by lateral instability are provided by Sakimoto & Komatsu (1982). They suggested the use of column curve *c* of the ECCS multiple column curves or similar curves. Practical applications for actual steel bridges are also presented (e.g., Sakimoto & Sakata 1990). Flexural-torsional buckling of arches has also been studied (e.g., Bradford & Pi 2006; Papangelis & Trahair 1987).

It is interesting to note that the arch bridges are amongst the oldest man-made bridges. The first use of cast-iron for a sizable structure resulted in the construction of the 30-m span Coalbrookdale arch bridge over the river Severn in the year 1779 in England. Several steel railroad and highway arch bridges have been constructed all over the world (e.g., the 503-m long Sydney Harbour bridge, Sydney, Australia, built in 1932 and designed by Freeman Fox, the 255-m long continuous tied arch bridge in Duisburg, Germany built in 1950, the 228-m long Fort Pitt, double deck tied arch bridge, Pittsburg, constructed in the 1950s, the New River Gorge Bridge, West Virginia, USA with a central span of 518 m (world's longest arch bridge built in 1977) and the 216-m long Changjiang river (railway) bridge at Juijiang, China built in the year 1992). The introduction of curved structural sections has allowed more creative bridge designs to be built. Steel arches could span about 20 to 90 m. For greater span (240 – 300 m), the arches may be made up of truss work. Even pre-stressed arch steel bridges have been built (Troitsky 1990; Nazir 2003). The advent of cable stayed bridges has somewhat reduced the use of arch bridges, since the segments of arch bridges have to be erected carefully with the use of temporary supports.

9.17.4 Composite/Cased Columns

Steel-concrete composite columns are being used increasingly in building structures, since they advantageously combine the properties of concrete (strong in compression, greater rigidity) with those of structural steel. They may be broadly classified as those with rolled and built-up shapes encased in concrete (see Fig. 1.46a of Chapter 1) and those with concrete placed inside a steel tube or SHS/RHS (see Fig. 1.46b of Chapter 1). Steel columns are sometimes cased in concrete for fire protection. Concrete encased columns can be either square or rectangular in cross section. A composite column with an I-section embedded in concrete and provided with regular reinforcement at the corners is shown in Fig. 1.46(a) in Chapter 1. The vertical reinforcement should be provided with lateral ties at regular intervals as per the concrete code (IS: 456-2000). The use of such composite columns results in high load-carrying capacity. In the design of composite columns, it is assumed that steel and concrete work together in resisting the loads. The steel section must comprise at least 4% of the total cross-sectional area, otherwise the columns must be designed as ordinary reinforced concrete columns.

Unfortunately, both IS 800:2007 and IS: 11384-1985 do not have provisions for the design of composite columns. The design of cased columns discussed here is based on BS 5950. The following conditions must be fulfilled while using the empirical method suggested by BS 5950-1:2000 (see also Fig. 9.52).

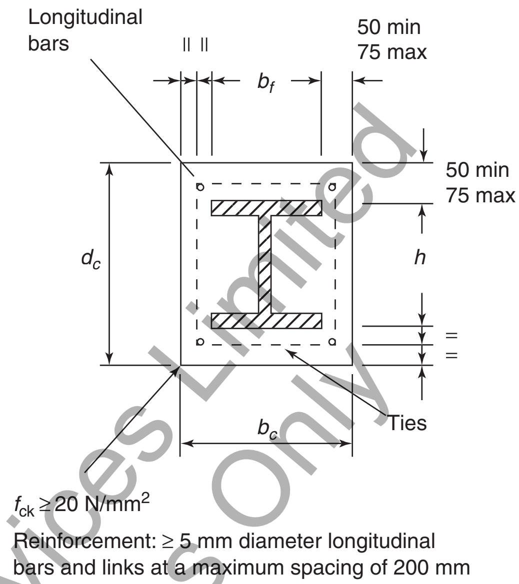


FIG. 9.52 Cased column—conditions specified in BS 5950-1:2000

1. The steel section should be either single rolled or fabricated I- or H- section with equal flanges. A pair of rolled channels in contact back-to-back or separated back-to-back by not less than 20 mm nor more than half their depth.
2. The overall dimensions of the steel section should not exceed 1000 mm (depth) × 500 mm (width).
3. The enclosing concrete should have a minimum characteristic compressive strength of f_{ck} = 20 MPa and should have a cover of more than 50 mm. The concrete should be thoroughly compacted, especially below cleats, cap plates, and beam soffits.
4. The ties around longitudinal bars should be more than 5 mm in diameter and their spacing should not exceed 200 mm.

The design procedure for axially loaded cased column is done as follows.

1. The effective length L_e is taken as the lesser of
   (a) $40\, b_c$
   (b) $\dfrac{100 b_c^2}{d_c}$
   (c) $250\, r$

   where b_c and d_c are the breadth and depth of the cased column (see Fig. 9.52), r is the minimum radius of gyration of the steel section alone, that is, r_y
2. The radius of gyration of the cased column about minor y-y axis, r_y, is taken as $0.2b_c$, but not more than $0.2\,(b_f + 150)$ mm, and not less than that of the steel section alone.
3. The radius of gyration of the cased column about major z-z axis, r_z, is taken as the radius of gyration of the steel section alone.

$f_{cd} = 133.25$ N/mm^2

Hence design strength = $f_{cd} \times A = 133.25 \times 85{,}000/1000 = 11{,}326$ kN > 11,000 kN

Hence, the section is suitable.

(b) The heaviest rolled section is wide flange W 610 × 320 with the following properties:

$A = 47630$ mm^2, $I_y = 30200 \times 10^4$ mm^4, $r_y = 79.6$ mm

Hence we have to provide substantial cover plates to make the area about 85,000 mm^2. As a first trial use 400 × 60 mm plates on both flanges as shown in Fig. 9.55(b).

$A = 47630 + 2\,(400 \times 60) = 95630$ mm^2

$I_y = 30200 \times 10^4 + 2\,(60 \times 400^3)/12 = 94200 \times 10^4$ mm^4

$r_y = \sqrt{(94200 \times 10^4/95{,}630)} = 99.25$ mm

From Table 10 of the code, since $t_f > 40$ mm use curve '*d*'.

$$\lambda = KL/r_y = 1 \times 8000/99.25 = 80.60$$

From Table 9d, for

$f_y = 230$ N/mm^2 and $\lambda = 80.6$,
$f_{cd} = 112.28$ N/mm^2

Capacity of the section = $112.28 \times 95630/1000$
= 10737 kN ≈ 11,000 kN

Hence the section is suitable.

Example 9.6: The framing plan of a multi-storey building is given in Fig. 9.56. Assume that all the columns have a size of ISHB 400; the longitudinal beams (z-direction) have a size of ISMB 600 and the transverse beams (y-direction) have a size of ISMB 400. The storey height is 3.5 m, and the columns are assumed to be fixed at the base. For a column in a typical lower floor of the building, determine the effective length KL_z and KL_y. For the purpose of estimating the total axial loads on the columns in the storey, assume a total distributed load of 35 kN/m^2 from all the floors above (combined). Assume Fe 410 grade steel.

Solution:

Unsupported length of the column, in *y*-direction,

$$L = 3500 - 400 = 3100 \text{ mm}$$

In *z*-direction, $L = 3500 - 600 = 2900$ mm

(a) *Relative stiffness of columns and beams*

ISHB 400: I_z of column = 28100×10^4 mm^4; $I_y = 2730 \times 10^4$ mm^3;

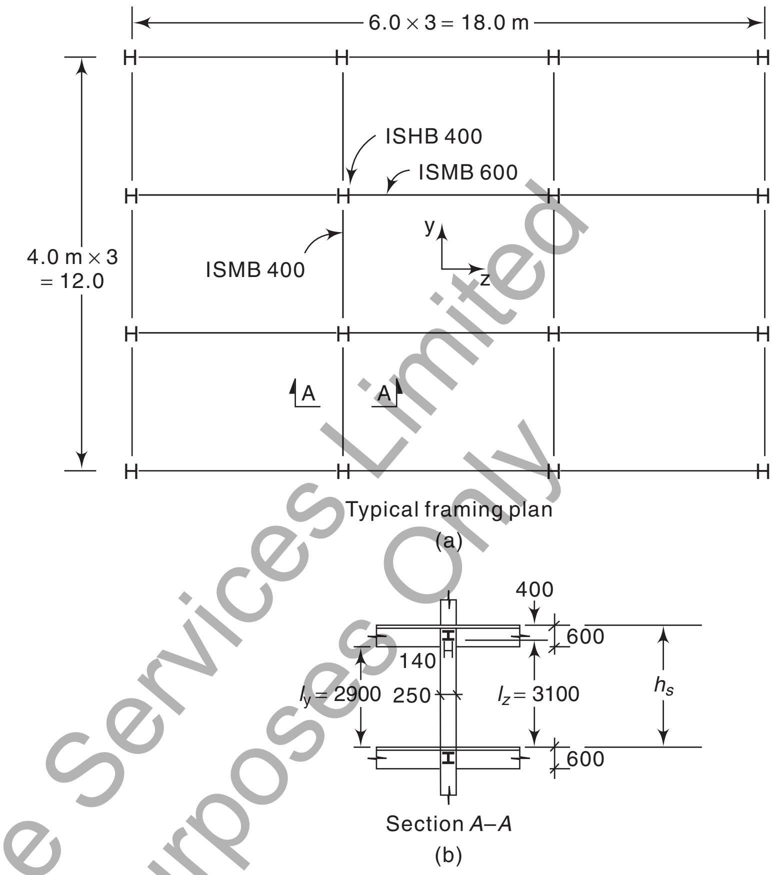


FIG. 9.56

$r_z = 169$ mm; $r_y = 52.6$ mm

ISMB 400: $I_z = 20500 \times 10^4$ mm^4; $I_y = 622 \times 10^4$ mm^4
ISMB 600: $I_z = 91800 \times 10^4$ mm^4; $I_y = 2650 \times 10^4$ mm^4

- Columns, 16 in number, ISHB 400, $h_s = 3500$ mm
  In *z*-direction

  $\Sigma I_c/h_s = 16 \times 28100 \times 10^4/3500 = 1284.5 \times 10^3$ mm^3

  In *y*-direction = $16 \times 2730 \times 10^4/3500 = 124.8 \times 10^3$ mm^3
- Longitudinal beams, 12 Nos, ISMB 600, $L_b = 6000$ mm

  $\Sigma(I_b/L_b)_z = 12 \times 91800 \times 10^4/6000 = 1836 \times 10^3$ mm^3
- Transverse beams

$$\Sigma(I_b/L_b)_y = 12 \times 20500 \times 10^4/4000 = 615 \times 10^3 \text{ mm}^3$$

(b) *Determination of whether column is braced or unbraced*
Ignoring the contribution of in-fill walls, as per Eqn (9.46)

$$\Delta_u/H_u = h_s^2/[12E\Sigma(I_c/h_s)] + 12E\Sigma(I_b/L_b)]$$

In longitudinal direction:

$$(\Delta_u/H_u) = 3500^2/[(12 \times 2 \times 10^5 \times 1284.5 \times 10^3) + (12 \times 2 \times 10^5 \times 1836 \times 10^3)] = 163.57 \times 10^{-8} \text{ mm/N}$$

EXAMPLE 9.8: For the unbraced frame shown in Fig. 9.58, calculate the effective length factor K for the columns in lines 2 and 3 and between second and third floor, using alignment chart and Wood's curve and compare the results.

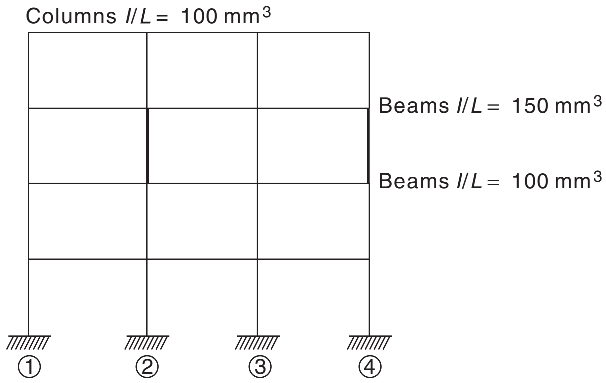


FIG. 9.58

SOLUTION:

(a) Using Julian and Lawrence Nomograph

For the columns of lines 2 and 3, we have

$$G_A = [(100 + 100)/(150 + 150)] = 0.67$$
$$G_B = [(100 + 100)/(100 + 100)] = 1$$

From Fig. 9.33, $K \approx 1.28$

(b) Using Wood's Curves

$$\beta_1 = \{(100 + 100)/[100 + 100 + 1.5(150 + 150)]\} = 0.308$$
$$\beta_2 = \{(100 + 100)/[100 + 100 + 1.5(100 + 100)]\} = 0.4$$

From Fig. 9.34, $K = 1.28$

The preceding value can be obtained from the formula given in the code as

$$K = \{[1 - 0.2\ (\beta_1 + \beta_2) - 0.12\beta_1\beta_2]/[1 - 0.8(\beta_1 + \beta_2) + 0.6\beta_1\beta_2]\}^{0.5}$$
$$= [(1 - 0.2 \times 0.708 - 0.12 \times 0.308 \times 0.4)/(1 - 0.8 \times 0.708 + 0.6 \times 0.308 \times 0.4)]^{0.5}$$
$$= 1.289$$

Thus, by using either method, we get the same result.

EXAMPLE 9.9: Calculate the compressive resistance of the leg of a transmission line tower consisting of a 200 × 200 × 20 angle section of height 3.0 m. Assume that the conditions at both ends of the *z-z* and *y-y* planes are such as to provide simple support. Take the design strength of steel as 250 N/mm² and assume that the load is concentrically applied to the angle.

SOLUTION:

Since the principal axes for an angle section do not coincide with the rectangular *y-y* and *z-z* axes, the buckling strength about the minor principal axis *v-v* should normally be checked. From Table 9.4, the curve *c* is appropriate for buckling about any axis. Therefore, only the axis about which the slenderness is greatest needs to be considered.

Check for limiting thickness

$$d/t = b/t = 200/20 = 10 < 15.7$$
$$(b + d)/t = 400/20 = 20 < 25.$$

Hence the section is not slender.

From section tables,

$$\text{area} = 7640\ \text{mm}^2$$
$$r_{xx} = r_{yy} = 61.4\ \text{mm},\ r_{vv} = 39.3\ \text{mm},$$

Maximum $\lambda = 3000/39.3 = 76.34$

From Table 9c of the code, for

$\lambda = 76.34$ and $f_y = 240$ MPa, (f_y reduced as per Table 1 of the code)

$$f_{cd} = 138.49\ \text{N/mm}^2$$

Hence design strength = 138.49 × 7640/1000 = 1058 kN

EXAMPLE 9.10: Calculate the compressive resistance of a 200 × 200 × 20 angle assuming that the angle is loaded through only one leg, when

(a) it is connected by two bolts at the ends
(b) it is connected by one bolt at each end
(c) it is welded at each end

Assume that the member has a length of 3 m and $f_y = 250$ MPa.

SOLUTION:

(a) Connected by two bolts at the ends

(i) For two bolts at each end, for fixed condition,

$k_1 = 0.20$, $k_2 = 0.35$, and $k_3 = 20$ (from Table 12 of the code)

$$\varepsilon = (250/f_y)^{0.5} = 1.0$$
$$\lambda_{vv} = (3000/39.3)/\sqrt{(\pi^2 E/250)} = 0.859$$
$$\lambda_\phi = (200 + 200)/\left[(2 \times 20)\sqrt{\pi^2 \times 2 \times 10^5/250}\right] = 0.1125$$
$$\lambda_e = [k_1 + k_2\lambda_{vv}^2 + k_3\lambda_\phi^2]$$
$$= \sqrt{[0.20 + 0.35 \times 0.859^2 + 20 \times 0.1125^2]} = 0.843$$

From Table 10 of the code, select buckling curve 'c' from Table 7 of the code, $\alpha = 0.49$.

$$f_{cd} = (f_y/\gamma_{mo})/[\phi + (\phi^2 - \lambda_e^2)^{0.5}]$$
$$\phi = 0.5\ [1 + 0.49(0.843 - 0.2) + 0.843^2] = 1.013$$
$$f_{cd} = (240/1.1)/[1.013 + (1.013^2 - 0.843^2)^{0.5}]$$
$$= 138.56\ \text{N/mm}^2$$
$$P_d = 138.56 \times 7640/1000 = 1058.6\ \text{kN}$$

(ii) If we consider the fixity condition as hinged (From Table 12 of the code)

$k_1 = 0.70$, $k_2 = 0.60$, and $k_3 = 5$

Hence $\lambda_e = \sqrt{(0.70 + 0.60 \times 0.859^2 + 5 \times 0.1125^2)}$

$= 1.098$

$\phi = 0.5\ [1 + 0.49(1.098 - 0.2) + 1.098^2] = 1.322$

$f_{cd} = (240/1.1)/[1.322 + (1.322^2 - 1.098^2)^{0.5}]$

$= 218.18/2.058 = 106\ \text{N/mm}^2$

$P_d = 106 \times 7640/1000 = 809.8$ kN

(b) Connected by only one bolt at the ends and the fixity condition taken as hinged,

$k_1 = 1.25,\ k_2 = 0.50,\ \text{and}\ k_3 = 60$

Hence $\lambda_e = \sqrt{(1.25 + 0.50 \times 0.859^2 + 60 \times 0.1125^2)}$

$= 1.542$

$\phi = 0.5[1 + 0.49(1.542 - 0.2) + 1.542^2] = 2.018$

$f_{cd} = (240/1.1)/[2.018 + (2.018^2 - 1.542^2)^{0.5}]$

$= 218.18/3.32 = 65.72\ \text{N/mm}^2$

$P_d = 65.72 \times 7640/1000 = 502$ kN

(c) When the strut is welded at each end, it is similar to case (a) and the strength will be equal to 1058.6 kN.

Note that when two bolts are provided at the ends, depending on the assumed fixity condition we get the capacity as 1058.6 kN or 809.8 kN. The value of 1058.6 kN is very close to the concentric loading case of Example 9.9, i.e., 1058 kN.

Let us cross-check this value with the Eqn (9.60).

$\lambda = (KL/r\pi)\sqrt{(f_y/E)}$

$= [3000/(39.3 \times \pi)]\ \sqrt{(250/2 \times 10^5)}$

$= 0.859$

$P_d = Af_y\ (0.990 + 0.150\lambda - 0.360\lambda^2 - 0.020\lambda^3)/\gamma_{m0}$

$= 250 \times 7640(0.990 + 0.15 \times 0.859 - 0.360 \times 0.859^2 - 0.02 \times 0.859^3)/(1.1 \times 1000)$

$= 191.03 \times 7640/1000 = 1459.5$ kN

This shows that the value adopted by the Indian code is very conservative.

Example 9.11: A part of an office floor and the elevation of internal column stack A are shown in Figs 9.59(a) and (b). The roof and floor loads are given as follows:

Roof: Dead load (total) = 5 kN/m^2

Imposed load = 1.5 kN/m^2

Floors: Dead load (total) = 5 kN/m^2

Imposed load = 4 kN/m^2

Design column A for axial load only. The self weight of the column, including fire protection, may be taken as 1 kN/m. The roof and floor steel have the same layout. Use Fe 410 grade steel. The slabs for the floor and roof are pre-cast one way spanning slabs.

Solution:

When calculating the load on column lengths, the imposed load may be reduced as discussed in Chapter 3.

Step 1: *Loading*

Four floor beams are supported at column A. These are designated as *B*1 and *B*2 in Fig. 9.59(a), the reaction from these beams in terms of a uniformly distributed load are shown in Fig. 9.59(c).

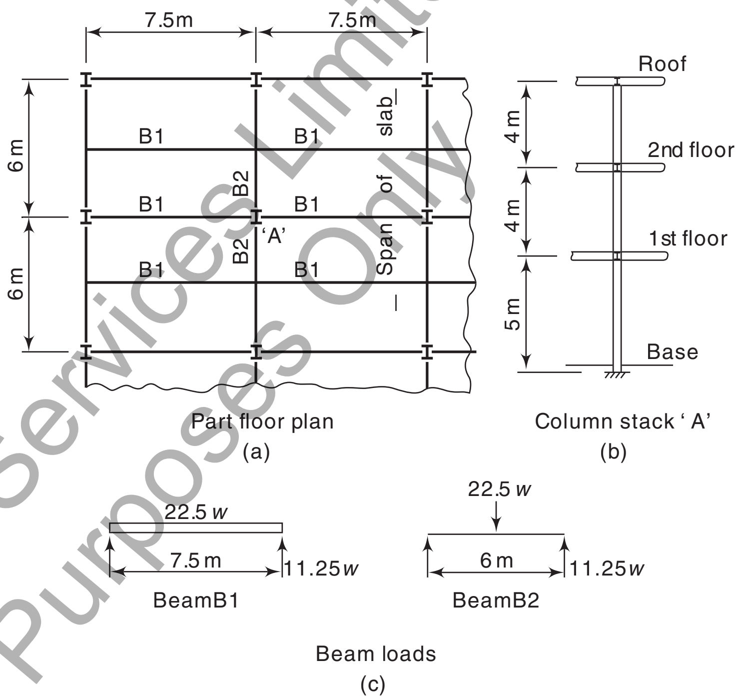


FIG. 9.59

Load on beam $B1 = 7.5 \times 3 \times w = 22.5\ w$ kN

where *w* is the uniformly distributed load. The dead and imposed loads are calculated separately to apply the reduction in imposed loads as suggested by IS 875. (This type of calculation may be useful when different load factors are applied to dead and imposed loads.) The self-weight of beam *B*2 is included in the reaction from beam *B*1. The loading calculations are shown in a tabular form as follows.

Dead load, kN	Imposed load, kN, Reduction			Total design load, kN
	0%	10%	20%	
(a) Column between roof and 2nd floor				
$w = 5$ kN/m^2	$w = 1.5$ kN/m^2			
Two nos *B*1: 112.5	33.75			
Two nos *B*2: 112.5	33.75			
Self wt of				
Column: 4.0				
Total: 229.0	67.5			

(*Contd*)

$A = 7480$ mm^2; $r_z = 130$ mm; $r_y = 54.1$ mm; $h/b = 300/250 = 1.2$, $t_f = 10.6$ m

$$L/r_z = 6000/130 = 46.15 < 180. \text{ Hence ok.}$$

$$L/r_y = 4000/54.1 = 73.94 < 180. \text{ Hence ok.}$$

From Tables 10 and 9b of the code, for

$$L/r_y = 73.94 \text{ and } f_y = 250 \text{ MPa}$$

$$f_{cd} = 159.7 \text{ N/mm}^2$$

Therefore, strength of the column = 159.7 × 7480/1000 = 1194.5 kN > 1100 kN

Hence, the chosen section is safe.

Note: If the beams in this example are rigidly connected to the columns, then the Wood's curves should be used to determine the effective length factor *K*.

EXAMPLE 9.13: The strut of a space frame member having a length of 3 m has to carry a factored load of 230 kN. Assuming $f_y = 220$ N/mm^2, design a circular tube to carry the load. Assume that the ends are simply supported.

SOLUTION:

Assume medium class pipe of outside diameter 114.3 mm with thickness 4.5 mm, weight = 121 N/m, $r = 38.9$ mm, and area = 1550 mm^2.

(a) *Check for limiting width-to-thickness ratio (see Table 8.7)*

$$D/t = 114.3/4.5 = 25.4 < 88\varepsilon^2 \; [88 \times (250/220)^2 = 93.8]$$

Hence, the tube is not slender.

(b) *Buckling curve classification*

From Table 10 of the code, for hot-rolled tubes, use buckling curve '*a*'.

(c) *Resistance to flexural buckling*

$$KL/r = 1 \times 3000/38.9 = 77.12$$

From Table 9a of the code,
for $f_y = 220$ N/mm^2 and $KL/r = 77.12$,

$$f_{cd} = 157.45 \text{ N/mm}^2$$

Design strength = $A \times f_{cd}$ = 1550 × 157.45/1000 = 244 kN > 230 kN

Hence, the assumed section is safe.

EXAMPLE 9.14: Repeat the problem given in Example 9.13 using a square tube, with $f_y = 240$ N/mm^2.

SOLUTION:

Assume a SHS of size 91.5 × 91.5 mm with a thickness of 4.5 mm, area = 1514 mm^2, weight = 118.8 N/m, $r = 35.2$ mm, and $\varepsilon = (250/f_y)^{0.5} = (250/240)^{0.5} = 1.042$

(a) *Check for limiting width-to-thickness ratio (see Table 8.7)*

$$D/t = 91.5/4.5 = 20.33 < 42\,\varepsilon (42 \times 1.042 = 43.7)$$

Hence, the section is not slender.

(b) *Buckling curve classification*

From Table 10 of the code, for hot-rolled tubes, use buckling curve '*a*'.

(c) *Resistance to flexural buckling*

$$KL/r = 1 \times 3000/35.2 = 85.3$$

From Table 9a of the code, for

$$f_y = 240 \text{ N/mm}^2 \text{ and } KL/r = 85.3$$

$$f_{cd} = 153.99 \text{ N/mm}^2$$

Hence design strength = 153.99 × 1514/1000 = 233.14 kN > 230 kN

Hence the assumed section is safe.

EXAMPLE 9.15: Design a double angle discontinuous strut to carry a factored load of 175 kN. The length of the strut is 3.0 m between intersections. The two angles are placed back-to-back and are tack bolted. Consider the following cases.

(a) Angles are placed on opposite sides of the gusset plate.
(b) Angles are placed on the same side of the gusset plate.
(c) Two angles in star formation.

Assume grade Fe 410 steel with $f_y = 250$ MPa.

SOLUTION:

Let us assume the design compressive stress is 0.4 f_y = 100 MPa.

Required area = 175 × 1000/100 = 1750 mm^2

(a) *Angles placed on opposite sides of the gusset plate*

Let us try 2 ISA 70 × 70 × 8 mm at 83 N/m with $A = 2 \times 1060$ mm^2,
$I_y = 47.4 \times 10^4$ mm^4, $r_z = 21.2$ mm, $c_y = 20.2$ mm

$$A^* = 2 \times 1060 = 2120 \text{ mm}^2$$

Assuming a 10-mm thick gusset plate

$$I_{yy}^* = 2[I_y + A(c_y + t_g/2)^2]$$
$$= 2[47.4 \times 10^4 + 1060(20.2 + 5)^2]$$
$$= 229.42 \times 10^4 \text{ mm}^2$$

$$r_y^* = \sqrt{(229.42 \times 10^4 / 2120)} = 32.90 \text{ mm}$$

$$r_z^* = r_z = 21.2 \text{ mm}$$

Note: The asterisk sign * is used for the properties of double angle strut.

Minimum radius of gyration

$$r^* = r_z = 21.2 \text{ mm}$$

Effective length factor could be between 0.7 and 0.85 (clause 7.5.2.1)

Assume $K = 0.85$.

Hence, effective length = 0.85 × 3000 = 2550 mm

L/r = 2550/21.2 = 120.28 < 180. Hence, the selected section is ok.

For L/r = 120.28 and f_y = 250 MPa, using Tables 10 and 9c of the code,

$$f_{cd} = 83.44 \text{ N/mm}^2$$

Hence strength of the member = 83.44 × 2120/1000 = 176.89 kN > 175 kN

Note: ISA 70 × 70 × 8 is not commonly rolled and hence the designer may choose 75 × 75 × 8 (89 N/m) or 80 × 80 × 6 (73 N/m), r = 24.6 mm

For 80 × 80 × 6, L/r = 103.65, f_{cd} = 102.47 MPa

Hence capacity = 102.47 × 2 × 929/1000 = 190.4 kN.

Hence, provide 2 ISA 80 × 80 × 6 as shown in Fig. 9.60(a).

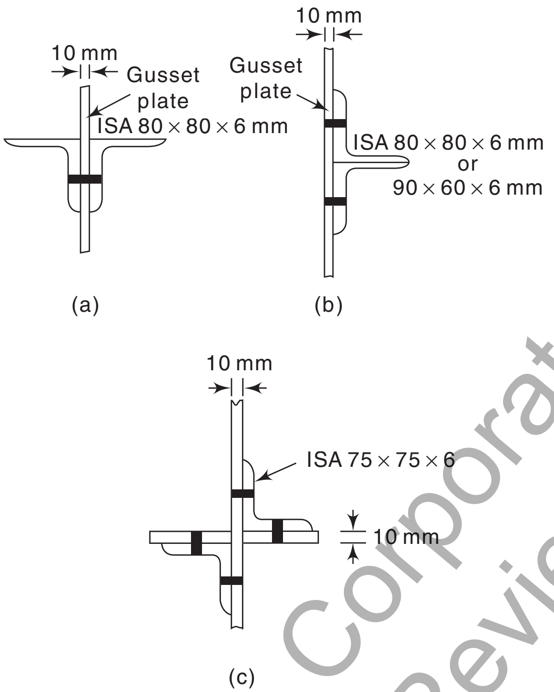


FIG. 9.60

(b) *Angles placed on the same side of the gusset plate*

Try 2L 75 × 75 × 6 mm at 68 N/m with A = 866 mm²; r_z = 23 mm; c_y = 20.6 mm;

$$I_z = I_y = 45.7 \times 10^4 \text{ mm}^4$$

r_z for two angles combined is the same as for one angle, i.e., 23 mm

$$A^* = 2 \times 866 = 1732 \text{ mm}^2$$

$$I^*_{yy} = 2 \times 45.7 \times 10^4 + 2 \times 866 \times 20.6^2 = 164.9 \times 10^4 \text{ mm}^4$$

$$r^*_y = \sqrt{(I_{yy}/A)} = \sqrt{[164.9 \times 10^4/(2 \times 866)]} = 30.8 \text{ mm}$$

Minimum radius of gyration $r^* = r_z$ = 23 mm

Effective length = 0.85 × 3000 = 2550 mm

L/r = 2550/23 = 110.87 < 180. Hence, the chosen section is safe.

For L/r = 110.87 and f_y = 250 MPa, from Table 10 and Table 9c of the code,

$$f_{cd} = 93.65 \text{ N/mm}^2$$

Hence capacity = 93.65 × 1732/1000 = 162.2 kN < 175 kN.

Hence try 80 × 80 × 6 at 73 N/m, with A = 929 mm² and r_z = 24.6 mm.

Now, L/r = 2550/24.6 = 103.66.

From Table 10 and Table 9c of the code, for L/r = 103.66 and f_y = 250 MPa,

$$f_{cd} = 102.47 \text{ MPa}$$

Capacity = 102.47 × 929 × 2 = 190.4 kN > 175 kN. Hence the assumed angle is safe.

Let us try an unequal angle of size 90 × 60 × 6 at 68 N/m, with 10 mm gap,

with A = 865 mm², r_z = 28.6 mm, r_y = 17.1 mm, I_y = 25.2 × 10⁴ mm⁴,

$$c_y = 13.9 \text{ mm}.$$

Radius of gyration about z-z axis for the combination is same as for one angle

$$r^*_z = 28.6 \text{ mm}$$

Radius of gyration of r_y is obtained by finding the moment of inertia about y-y axis

$$I^*_{yy} = 2 \times 25.2 \times 10^4 + 2 \times 865\ (13.9 + 10/2)^2$$
$$= 50.4 \times 10^4 + 61.79 \times 10^4$$
$$= 112.19 \times 10^4 \text{ mm}^4$$
$$r^*_y = \sqrt{[112.19 \times 10^4/(2 \times 865)]} = 25.46 \text{ mm}$$

Hence r^*_{min} = 25.46 mm

$$L/r = 2550/25.46 = 100.13$$

From Tables 10 and 9c of the code for L/r = 100.13 and f_y = 250 MPa,

$$f_{cd} = 106.84 \text{ MPa}$$

Capacity = 106.84 × 2 × 865/1000 = 184.83 kN > 175 kN

Hence, the section is safe.

Note: From these calculations, it is seen that the designers' task is to design an angle that will be available in the market and also to choose a section that will provide an optimum section. It is clear that two unequal angles with the longer leg connected to the gusset plate provides the economic solution. Two angles on the opposite side of the gusset plate will be economical than two angles on the same side of the gusset plate. Tack welding at a spacing of 550 mm (using stitch plates) along the length has to be provided to avoid local buckling of the individual angles [see calculation in part (c)].

(c) *Star angle*

Choose two angles 75 × 75 × 6 mm at 68 N/m in a star configuration as shown in Fig. 9.60(c). The relevant properties of single 75 × 75 × 6 angle at 68 N/m are

$A = 866$ mm^2; $r_z = r_y = 23$ mm; $c_z = c_y = 20.6$ mm; $r_u = 29.1$ mm; $r_v = 14.6$ mm

Assuming the gusset plate thickness as 10 mm,

$$A^* = 2 \times 866 = 1732 \text{ mm}^2$$

$$r_z^* = r_y^* \sqrt{[r^2 + (c_y + 10/2)^2]} = \sqrt{[23^2 + (20.6 + 10/2)^2]}$$

$$= 34.41 \text{ mm}$$

$$r_v^* = \sqrt{[r_v^2 + 2(c_y + 10/2)^2]} = \sqrt{[14.62^2 + 2(20.6 + 10/2)^2]}$$

$$= 39.04 \text{ mm}$$

$$r_u^* = 29.1 \text{ mm}$$

The least radius of gyration

$$r_{\min}^* = 29.1 \text{ mm} = r_u \text{ of single angle}$$

Hence $\lambda = L/r = 2550/29.1 = 87.62 < 180$

For $\lambda = 87.62$ and $f_y = 250$ MPa from Tables 10 and 9c of the code,

$$f_{cd} = 124.57 \text{ N/mm}^2$$

Capacity of the section = 124.57 × 1732/1000
= 215.75 kN > 175 kN.

The chosen section has a capacity much higher than the required strength and hence, a lower available angle section may be chosen. The reader may do it as an exercise and select a suitable angle.

Note that the star type configuration results in a smaller section compared to the other two angle configurations in (a) and (b). Such star angles are often used for the legs of transmission or communication towers (see Subramanian 1992). They are also used as bracings in industrial buildings.

Tack Welding

Tack welding along the length should be provided to avoid local buckling of each of the elements. The spacing of the welds to be adjusted based on (clause 7.8.1 of the code)

$$\lambda_e \le 0.6\lambda = 0.6 \times 87.62 = 52.05 \text{ or } \le 40$$

Selecting the lowest value,

$$\lambda_e = 40$$

The spacing of the tack weld is given by

$$\lambda_e = S/r_v \le 40$$

Hence $S = 40r_v = 40 \times 14.6 = 584$ mm

The welding should be designed to resist a transverse load of 2.5% of the axial load, i.e., 175 × 1000 × 2.5/100 = 4375 N.

Assuming a 3 mm fillet weld (as per Table 6.3),

Design strength of the weld (Table 6.5) = 158 N/mm^2

Hence length of weld = 4375/(0.7 × 3 × 158) = 13.2 mm

Hence provide a 3 mm tack welding of 15-mm length at 550 mm spacing (see Fig. 7.3 given in Chapter 7 for the details of stitch plates).

EXAMPLE 9.16: Design a single angle discontinuous strut to carry a factored load of 65 kN. Assume that the distance between its joints is 2.5 m and is loaded through one leg. Use $f_y = 250$ MPa.

SOLUTION:

Take effective length = L = 2500 mm

Assume a slenderness ratio of 120 and the corresponding f_{cd} = 83.7 MPa (from Table 9.3)

$$\text{Area required} = 65 \times 1000/83.7 = 777 \text{ mm}^2$$

Choose ISA 75 × 75 × 6 at 68 N/m with

$$A = 866 \text{ mm}^2,\ r_{vv} = 14.6 \text{ mm}$$

Check for section classification (Table 2 of the code)

$$d/t = b/t = 75/6 = 12.5 < 15.7\varepsilon$$

Hence the section is semi-compact

$$(b + d)/t = 150/6 = 25 \le 25\varepsilon$$

Hence take effective area as gross area.

Assuming two bolts at each end and fixed condition (from Table 12 of the code)

$$k_1 = 0.2,\ k_2 = 0.35, \text{ and } k_3 = 20$$

$$\varepsilon = (250/f_y)^{0.5} = 1.0$$

$$\lambda_{vv} = (L/r_{vv})/\left[\varepsilon\sqrt{(\pi^2 E/250)}\right]$$

$$= (2500/14.6)/\left[1.0 \times \sqrt{(\pi^2 \times 2 \times 10^5/250)}\right]$$

$$= 171.23 \times 0.01125 = 1.926$$

$$\lambda_\phi = (b_1 + b_2)/\left[\varepsilon\sqrt{(\pi^2 E/250)} \times 2t\right]$$

$$= 150 \times 0.01125/(2 \times 6) = 0.1406$$

$$\lambda_e = \sqrt{[k_1 + k_2\lambda_{vv}^2 + k_3\lambda^2]}$$

$$= \sqrt{[0.2 + 0.35 \times 1.926^2 + 20 \times 0.1406^2]} = 1.376$$

$$f_{cd} = (f_y/\gamma_{mo})/[\phi + (\phi^2 - \lambda_e^2)^{0.5}]$$

$$\phi = 0.5[1 + 0.49(1.376 - 0.2) + 1.376^2] = 1.735$$

$$f_{cd} = (250/1.1)/[1.735 + (1.735^2 - 1.376^2)^{0.5}]$$

$$= 81.4 \text{ N/mm}^2$$

$$P_d = 81.4 \times 866/1000 = 70.497 \text{ kN} > 65 \text{ kN}$$

Hence, the assumed section is safe.

Note: If the ends are hinged, the capacity is 36 kN only.

EXAMPLE 9.17: Design the single angle section of Example 9.16 assuming that it is concentrically loaded.

Spacing of channels

$$2I_{zz} = 2[I_{yy} + A(S/2 + c_{yy})^2]$$

Thus, $2 \times 6420 \times 10^4 = 2[313 \times 10^4 + 4630(S/2 + 23.5)^2]$
or $(S/2 + 23.5)^2 = 13190$

$$S = 182.70 \text{ mm}$$

Let us keep the channels at a spacing of 183 mm.
Lacing system Using a single lacing system with the inclination of lacing bar = 45° (gauge length for a 90 mm flange = 50 mm)
Spacing of lacing bars, $L_o = 2(183 + 50 + 50) \cot 45° = 2 \times 283 \times 1 = 566$ mm
L_o/r_{yy} should be $< 0.7 \times L/r$ of whole column
$21.77 < 0.7 \times 88.98 = 62.3$
Hence safe.
Maximum shear = $(2.5/100) \times 1100 \times 10^3 = 27{,}500$ N
Transverse shear in each panel = $V/N = 27{,}500/2 = 13{,}750$ N
Compressive force in the lacing bar = (V/N) cosec 45°
= $13750 \times 1.414 = 19445$ N
Assuming 16-mm diameter bolts,
Minimum width of lacing flat (clause 7.6.2 of the code) = 3×16, say 50 mm
Minimum thickness = (1/40)(183 + 50 + 50) cosec 45° = 10.01 mm
Provide 12-mm thick plate with a width of 50 mm
Minimum $r = t/\sqrt{12} = 12/\sqrt{12} = 3.464$ mm
L/r of the lacing bar = $283 \times$ cosec 45°/3.464 = $115.5 < 145$
Hence safe.
For $L/r = 115.5$ and $f_y = 250$ MPa, using Table 9c of the code

$$f_{cd} = 88.6 \text{ MPa}$$

Load carrying capacity = $88.6 \times 50 \times 12 = 53{,}163$ N > 19,445 N
Hence the lacing bar is safe.
Tensile strength of the lacing flat = $0.9(B - d)tf_u/\gamma_{m1}$ or f_yA_g/γ_{m0}
Thus $0.9(50 - 18) \times 12 \times 410/1.25$ or $250 \times 50 \times 12/1.1$

113,356 N or 136,363

Thus, the tensile strength of the lacing flat = 113,356 N > 19,445 N
Hence the lacing flat is safe.
Check
r_{min} of the built-up column = 118 mm,

$$L/r = 10{,}000/118 = 84.74$$

r_{min} of the individual chords = 26.0 mm,

$$L_0/r = 566/26 = 21.77$$

λ of the built-up column [Eqn (9.63)],

$$\lambda_e = \sqrt{\{84.74^2 + 3.14^2(9260/600) \times 400.22^3/(566 \times 230)^2)\}}$$
$$= 86.64 < 88.98$$

Hence, the column is safe.

Connection: Assuming that the 16 mm bolts of grade 4.6 are connecting both the lacing flats with the channel at one point and that the shear plane will not pass through the threaded portion of bolt.
Strength of bolt in double shear = $2 \times A_{sb}(f_u/\sqrt{3})/\gamma_{mb}$

$$= 2 \times \pi \times 16^2/4 \times (400/\sqrt{3})/1.25 = 74{,}293 \text{ N}$$

Strength in bearing = $2.5k_bdtf_u/\gamma_m$ (with $k_b = 0.49$) = $2.5 \times 0.49 \times 16 \times 12 \times 410/1.25$
= 77,145 N
Hence, strength of bolt = 74,293 N > 19,445 N
Hence one 16-mm diameter bolt of grade 4.6 is required.
Tie plates: Tie plates must be provided at the ends of the laced column.
Effective depth = $183 + 2 \times c_{yy} > 2 \times b_f$
= $183 + 2 \times 23.5 = 230$ mm $> 2 \times 90$
= 180 mm
Hence
Required overall depth of tie plate = $230 + 2 \times 25 = 280$ mm
(edge distance of 16-mm diameter bolts = 25 mm)
Provide a tie plate of 300-mm depth.
Length of tie plate = $183 + 2 \times 90 = 363$ mm
Required thickness of tie plate = $1/50(183 + 2g) = 1/50(183 + 2 \times 50) = 5.66$ mm
(where g = gauge distance—see Fig. 5.10 of Chapter 5)
Hence, provide a tie plate of 6-mm thickness.
Provide a tie plate of size $363 \times 300 \times 6$ mm at both ends with six 16-mm diameter bolts.

(b) Consider the case of laced column with the two channels provided toe-to-toe.
Spacing (see Fig. 9.61b):

$$2I_{zz} = 2[I_{yy} + A(S/2 - c_{yy})^2]$$
$$2 \times 6420 \times 10^4 = 2[313 \times 10^4 + 4630(S/2 - 23.5)^2]$$
$$\text{or } (S/2 - 23.5)^2 = 13190$$
$$S = 276.7 \text{ mm}$$

Let us place the channel at a spacing of 280 mm.
Connecting system: Assuming single lacing system is provided with an inclination of 45°; gauge length for 90 m flange = 50 mm

$$L_o = 2(280 - 50 - 50) \cot 45° = 360 \text{ mm}$$
$$L_o/r_{yy} = 360/26 = 13.8 < 50$$

Hence L_o/r_{yy} ratio is fine.
$0.7(L/r)$ of combined channel = $0.7 \times 88.98 = 62.3 > 13.8$
Hence the L/r ratio is ok.
Compressive force in lacing bar = 19,445 N
Minimum width of lacing flat for 16 mm bolt (clause 7.6.2 of the code) = 50 mm
Minimum thickness = $1/40(280 - 50 - 50) \times$ cosec 45°

= 6.36 mm

Hence, provide a 50 × 8 mm flat.

Check $r_{min} = t/\sqrt{12} = 8/\sqrt{12} = 2.309$ mm

$L/r = 180 \times \text{cosec } 45°/2.309 = 110.2 < 145$

Hence the chosen flat is safe.

For $L/r = 110.2$ and $f_y = 250$ MPa, from Table 9c of the code,

$$f_{cd} = 94.4 \text{ N/mm}^2$$

Capacity of the lacing flat = 94.4 × 50 × 8
= 37,760 N > 19,445 N

Tensile strength of the lacing flat = $0.9(B - d)tf_u/\gamma_{m1}$ or $f_y A_g/\gamma_{m0}$
= 0.9(50 − 18) × 8 × 410/1.25 or 250 × 50 × 8/1.1
= 75,571 N or 90,909 N both > 19,445 N

Hence the lacing flat is safe.

Connection:

Strength of bolt in double shear {from (a)} = 74,293 N

Strength in bearing = $2.5k_b dtf_u/\gamma_{mb}$ = 2.5 × 0.49 × 16 × 8 × 410/1.25 = 51,430 N

Hence, strength of bolt = 51,430 N > 19,445 N

Therefore, provide one 16-mm diameter bolt of grade 4.6.

Tie plates:

Effective depth of tie plate = $S - 2c_{yy}$
= 280 − 2 × 23.5 = 233 mm > 2 × 90 = 180 mm

Required overall depth = 230 + 2 × 25 = 280 mm (edge distance of 16-mm diameter bolt = 25 mm)

Provide a 300 mm plate

Length of tie plate = 280 mm

Thickness of tie plate = 1/50(280 − 2 × 50) = 3.6 mm

Provide 6 mm.

Provide a tie plate of size 280 × 300 × 6 mm and use six bolts of 16-mm diameter and grade 4.6 to connect it to the channels. The arrangement is shown in Fig. 9.61(b). It is seen that by providing channels toe-to-toe, the lacing size and the tie plate size are reduced.

(c) From part (a) of this example,

Spacing of the channels = 183 mm

Compressive force in the lacing = 19,445 N

Effective length of lacing flat (welded)
= 0.7 × 183 × cosec 45° = 181.16 mm

Minimum thickness of flat = 1/40 × (183 × cosec 45°)
= 6.47 mm

Provide 50 × 8 mm lacing flat.

Minimum radius of gyration, $r = t/\sqrt{12} = 8/\sqrt{12} = 2.31$ mm

$$L/r = 181.16/2.31 = 78.4 < 145$$

Hence the L/r ratio is ok.

For $L/r = 78.4$ and $f_y = 250$ MPa, using Table 9c of the code

$$f_{cd} = 138.56 \text{ N/mm}^2$$

Capacity of lacing bar = 138.56 × 50 × 8 = 55,424 N > 19,445 N

Hence the lacing bar is safe.

Overlap of lacing flat = 50 mm > 4 × 8 = 32 mm

Hence the lacing flat is safe.

Connection:

Thickness of flange of ISMC 300 = 13.6 mm

Minimum size of weld = 5 mm (Table 6.3)

Strength of weld/unit length = $0.7 \times 5 \times 410/(\sqrt{3} \times 1.5)$
= 552 N/mm

Required length of weld = 19445/552 = 35.2 mm

Adding extra length for ends, the weld length to be provided

= 36 + 2(2 × 5) = 56 mm

Provide 100 mm weld length at both ends.

Tie plate:

Overall depth of plate = $183 \times 2 \times c_{yy}$
= 183 + 2 × 23.5
= 230 mm > 2 × 90 mm

Let length of tie plate = 183 + 2 × 50 = 283 mm

Thickness of tie plate = 1/50(183 + 2 × 50) = 5.66 mm

Provide a 8 mm plate to accommodate a 5 mm weld.

Provide a tie plate of 283 × 240 × 8 mm size and connect it with 5 mm welds as shown in Fig. 9.61(c).

EXAMPLE 9.19: Design a batten system for the column in Example 9.18. Assume that the two channels are kept back-to-back. (See Fig. 9.62)

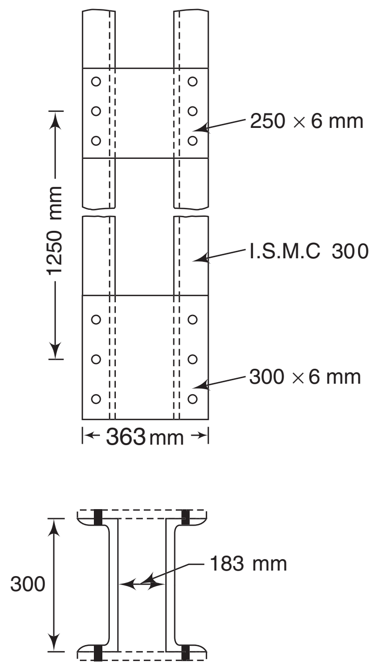


FIG. 9.62

$$f_{cd} = 193.71 \text{ MPa}$$

Capacity of the built-up column = $193.71 \times 2 \times 6380/1000 = 2471$ kN > 2400 kN

Hence the column is safe.

Spacing of channels:

$$2I_{zz} = 2[I_{yy} + A(S/2 + c_{yy})^2]$$

$$2 \times 15200 \times 10^4 = 2[508 \times 10^4 + 6380\ (S/2 + 24.2)^2]$$

$$(S/2 + 24.2)^2 = 23028$$

Hence $S = 255.1$ mm. Therefore, provide two ISMC 400 at a spacing of 256 mm back-to-back as shown in Fig. 9.63.

Spacing of battens:

L_o/r_{yy} should be less than $0.7 \times$ slenderness ratio of the built-up column. Hence,

$$L_o/r_{yy} < 0.7(L/r)$$

$$L_o < 0.7(L/r)r_{yy}$$

$$< 0.7 \times 42.86 \times 28.2$$

$$< 846 \text{ mm}$$

Also, L_o/r_{yy} should be less than 50. Therefore,

$$L_o/r_{yy} < 50$$

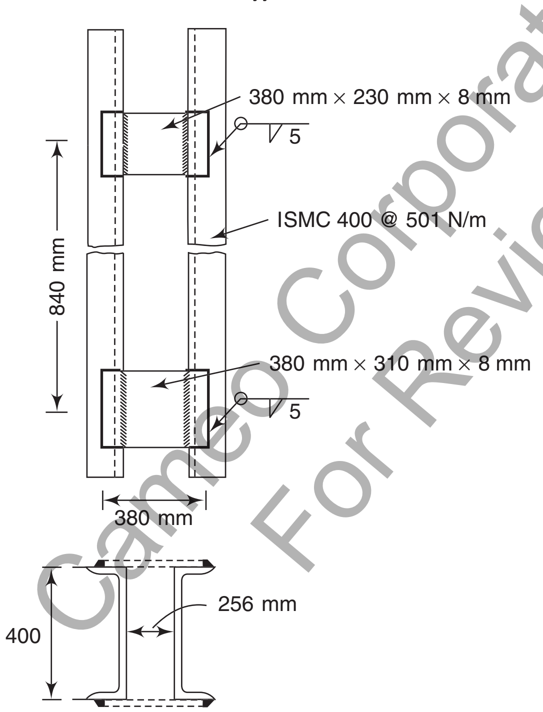


FIG. 9.63

i.e., $L_o < 50 \times 28.2 = 1210$ mm

Provide the batten at a spacing of 840 mm.

Size of end battens

Overall depth of batten = $256 + 2 \times c_{yy}$
$= 256 + 2 \times 24.2 = 304.4$ mm

Provide a 62 mm overlap of batten on channel flange for welding.

Length of batten = $256 + 2 \times 62 = 380$ mm

Thickness of batten = $1/50 \times 380 = 7.60$ mm

Provide $380 \times 310 \times 8$ mm end batten plate.

Size of intermediate battens

Overall depth = $3/4 \times 304.4 = 228.3$ mm

Provide $380 \times 230 \times 8$ mm intermediate battens.

Design forces:

Transverse shear = 2.5% of axial load
$= 2.5 \times 2400 \times 10^3/100 = 60{,}000$ N

Longitudinal shear $V_b = V_t L_o/ns$
$= 60{,}000 \times 840/[2(256 + 2 \times 62/2)]$
$= 79{,}245$ N

Moment $M = V_t L_o/2n$
$= 60{,}000 \times 840/(2 \times 2) = 12.6 \times 10^6$ N mm

Check

(a) For end battens,

Shear stress = $79{,}245/(310 \times 8)$
$= 31.9 \text{ MPa} < 250/(\sqrt{3} \times 1.1) = 131.2$ MPa

Bending stress = $12.6 \times 10^6 \times 6/(8 \times 310^2)$
$= 98.3 \text{ MPa} < 250/1.1 = 227$ MPa

(b) For intermediate battens,

Shear stress = $79{,}245/(230 \times 8)$
$= 43.1 \text{ MPa} < 131.2$ MPa

Bending stress = $12.6 \times 10^6 \times 6/(8 \times 230^2)$
$= 178.6 \text{ MPa} < 227$ MPa

Hence the battens are safe.

Design of weld:

Let the welding be done on all the four sides at each end of batten plate as shown in Fig. 9.63.

Let t be the throat thickness of weld.

$I_{zz} = 2[(62 \times t^3/12) + (62 \times t)(230/2)^2] + 2 \times t \times 230^3/12$

Neglecting $62 \times t^3/12$, which will be insignificant, we get

$$I_{zz} = 366.77 \times 10^4 t \text{ mm}^4$$

$$I_{yy} = 2[t \times 62^3/12] + 2 \times 230 \times t^3/12 + 2 \times 230 \times t \times 31^2$$

Again neglecting $2 \times 230 \times t^3/12$, which will be very small, we get

$$I_{yy} = 48.18 \times 10^4\ t \text{ mm}^4$$

$$I_p = I_{zz} + I_{yy} = t(366.77 \times 10^4 + 48.18 \times 10^4)$$

$$= 414.95 \times 10^4 t \text{ mm}^4$$

The same result may be obtained by using Table 6.7.

$$I_p = t\frac{(b+d)^3}{6} = t\frac{(62+230)^3}{6}$$

$$= 414.95 \times 10^4 t \text{ mm}^4$$

$$r = \sqrt{[(230/2)^2 + (62/2)^2]} = 119.1 \text{ mm}$$

$$\cos\theta = 31/119.1 = 0.260$$

Direct shear stress = 79245/(2 × 62 + 2 × 230)t
= 135.7/t N/mm^2

Shear stress due to bending moment
= 12.6 × 10^6 × 119.1/(414.95 × 10^4 × t)
= 361.65/t N/mm^2

Combined stress [clause 10.5.10 of the code]
$[3(135.7/t)^2 + (361.65/t)^2]^{0.5}$
= 431.2/t < 410/($\sqrt{3}$ × 1.25) = 189.4
or t = 431.2/189.4 = 2.28 mm

Size of weld = 2.28/0.7 = 3.26 mm

The size of weld should not be less than 5 mm for 15.3 mm flange. Hence, provide a 5 mm weld to make the connections.

EXAMPLE 9.21: Design a built-up laced column with four angles to support an axial load of 900 kN. The column is 12-m long and both the ends are held in position and restrained against rotation. Assume Fe 410 grade steel.

SOLUTION:

Required area of the column = 900 × 10^3/(0.6 × 250) = 6000 mm^2

Provide four angles ISA 100 × 100 × 8 mm. In this case we do not have any restriction of size (except for any architectural constraint). Hence, we will work backwards on the spacing from the assumed area and design compressive stress. The relevant properties are

$$A = 1540 \text{ mm}^2$$
$$c_{zz} = c_{yy} = 27.6 \text{ mm}$$
$$r_{zz} = r_{yy} = 30.7 \text{ mm}$$
$$I_{zz} = I_{yy} = 145 \times 10^4 \text{ mm}^4$$

Area provided = 4 × 1540 = 6160 mm^2

For 6280 mm^2, the required f_{cd} = 900 × 10^3/6160 = 146.1 MPa

From Table 9c, allowable L/r (for f_{cd} = 146.1 MPa) ≈ 75

From Table 11 of the code, for the fixed condition, K = 0.65

As it is a laced column

L = 1.05 × [0.65 × 12 × 10^3] = 8190 mm

Required r = 8190/75 = 109.2 mm

Moment of inertia of required section $I = Ar^2$
= 6160 × 109.2^2 = 73.46 × 10^6 mm^4

Equating required and provided moment of inertia,

$$73.46 \times 10^6 = 4 \times 145 \times 10^4 + 6160\ \bar{y}^2$$
$$\bar{y} = 104.8 \text{ mm}$$

Spacing of angle S = 2 × (104.8 + 27.6) = 264.8 mm

Therefore, provide S = 265 mm

Now, $I_{zz} = I_{yy}$ = 4 × 145 × 10^4 + 6160(265/2 − 27.6)^2 = 73.58 × 10^6 mm^4

$$r = \sqrt{(73.58 \times 10^6/6160} = 109.3$$
$$L/r = 8190/109.3 = 74.93$$

From Table 9c, for L/r = 74.93 and f_y = 250 MPa,

$$f_{cd} = 144.11 \text{ MPa}$$

Capacity of the built-up column = 6160 × 144.11/1000 = 887.72 kN ≈ 900 kN

Hence the column is safe. If necessary, the spacing may be increased to 270 mm.

Connecting system:

Let us provide a double lacing system with the lacing flats inclined at 45°. Both are provided at the centre of the leg of angle.

Spacing of lacing bar, L_o = (270 − 50 − 50)cot45°
= 170 mm

L_o/r_{yy} = 170/30.7 = 5.5 < 50

It should also be less than 0.7 × 74.9 = 52.43 > 5.5

Shear force, V = (2.5/100) × 900 ×10^3 = 22,500 N

Transverse shear in each panel = V/N = 22,500/2 = 11250 N

As double lacing is provided,

Compressive force in lacing bar = (V/2N)cosec θ
= 11250/2 × cosec 45°
= 7955 N

Section of lacing flat:

Assuming 20 mm bolts, width of the flat (clause 7.6.2) = 60 mm

Length of lacing = (270 − 50 − 50) cosec 45° = 240.4

Minimum thickness of the lacing flat = (1/60) × 240.4 (Since double lacing)
= 4.0 mm

Provide a flat of size 60 × 6 mm.

Minimum radius of gyration,

$$r = t/\sqrt{12} = 6/\sqrt{12} = 1.73 \text{ mm}$$

$$L_1/r = 0.7(240.4)/1.73 = 97.27 < 145$$

Hence the flat is safe.

For L_1/r = 97.27 and f_y = 250 MPa, from Table 9c,

$$f_{cd} = 110.82 \text{ N/mm}^2$$

Capacity of lacing bar = 110.82 × 60 × 6
= 39,895 N > 7955 N

Hence the lacing bar is safe.

Connections:

Strength of a 20 mm diameter bolt in double shear (Table 5.9) = 2 × 45.3 = 90.6 kN

Strength of the bolt in bearing = $2.5k_b dt \times f_u/\gamma_{mb}$
= 2.5 × 0.6 × 120 × 6 × 410/(1.25 × 1000) = 59 kN

Hence strength of bolt = 59 kN

Number of bolts = 2 × 7955 × cot45°/(59 × 10^3) = 0.27

Provide one 20-mm diameter bolt.

Note: The size of the lacing plate may be reduced to 50 × 6 m with a 16-mm diameter bolt.

Tie plate

Tie plates are to be provided at each end of the built-up column.

A = 1785 mm^2, r = 34.8 mm, and t = 5.4 mm

Classification of the cross section:

$$\varepsilon = \left(\frac{250}{f_y}\right)^{0.5} = \left(\frac{250}{310}\right)^{0.5} = 0.898$$

$$b = B - 3t = 91.5 - 3 \times 5.4 = 75.3 \text{ mm}$$

$$b/t = 75.3/5.4 = 13.94 < 42\varepsilon = 42 \times 0.898 = 37.7$$

Hence the section is non-slender.

Calculate design stress:

From Table 11 of the code, K = 2.0

$$KL/r = \frac{2 \times 2500}{34.8} = 143.7 < 180$$

Assuming hot rolled type, from Table 10 of the code, we need to use buckling curve *a*.

From Table 9a of the code, for L/r = 143.7 and f_y = 310 MPa, f_{cd} = 76.5 N/mm^2

Design capacity = 76.5 × 1785/1000 = 136.5 kN > 135 kN

Hence use SHS of size 91.5 mm × 91.5 mm medium class tube of thickness 5.4 mm.

EXAMPLE 9.25: Calculate the compressive resistance of the pin-ended ISMB 350 column of Example 9.2, having a length of 3 m, if it is encased in concrete of compressive strength 20 N/mm^2, as shown in Fig. 9.65.

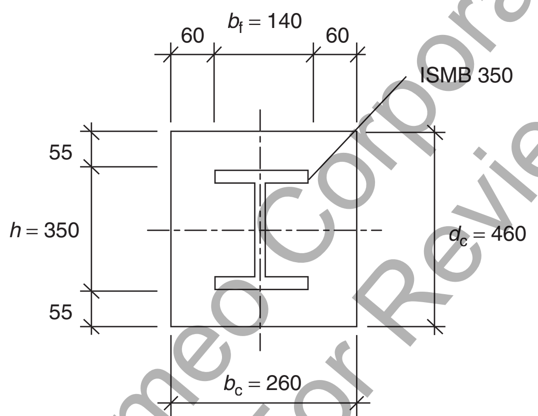


FIG. 9.65

SOLUTION:

Properties of ISMB 350

Flange width, b_f = 140 mm

Depth of section, h = 350 mm

Cross section area, A_g = 6670 mm^2

Radius of gyration, r_z = 143 mm, r_y = 28.4 mm

Area of concrete = $A_c = b_c \times d_c$ = 260 × 460 = 119,600 mm^2

Check for effective length

The effective length of column (3000 mm) should not exceed the least of:

1. $40b_c = 40 \times 260 = 10{,}400$ mm
2. $\frac{100b_c^2}{d_c} = \frac{100 \times 260^2}{460} = 14{,}696$ mm
3. 250 r_y = 250 × 28.4 = 7,100 mm

Hence it is OK.

Check for radii of gyration of cased section

For the cased section, r_z is the same as the steel section = 143 mm

For the cased section, $r_y = 0.2b_c$ = 0.2 × 260 = 52 mm < 0.2 (b_f + 150) = 0.2 (140 + 150) = 58 mm, but not less than that of the uncased section = 28.4 mm.

Hence r_y = 52 mm and r_z = 143 mm.

Compression resistance

Slenderness ratio

$\frac{KL_z}{r_z} = \frac{3000}{143} = 20.98$, from Tables 10 and 9(a) of the code, f_{cd} = 225. 4 MPa

$\frac{KL_y}{r_y} = \frac{3000}{52} = 57.7$, from Tables 10 and 9(b) of the code, f_{cd} = 184 MPa

Hence design axial compressive stress, , f_{cd} = 184 MPa

$$f_{ck} = 20 \text{ MPa}, f_y = 250 \text{ MPa}$$

The compression resistance of the cased column, P_{cd}, is determined as

$$P_{cd} = \left(A_g + \frac{0.45 f_{ck} A_c}{f_y}\right) f_{cd}$$

$$= \left(6670 + \frac{0.45 \times 20 \times 119{,}600}{250}\right) 184 \times 10^{-3}$$

$$= 2019.5 \text{ kN}$$

which should not be greater than the short strut capacity, P_{cs}, given by

$$P_{cs} = \left(A_g + \frac{0.25 f_{ck} A_c}{f_y}\right) f_{cd}$$

$$= \left(6670 + \frac{0.25 \times 20 \times 119{,}600}{250}\right) 250 \times 10^{-3}$$

$$= 2265.5 \text{ kN}$$

Hence the compression resistance of the cased section is 2019.5 kN. Comparing it with the compressive resistance of the uncased column (Example 9.2) shows that the load capacity of the column has been increased from 733.7 kN to 2019.5 kN due to the casing by concrete.

SUMMARY

A structural member subjected to compressive forces along its axis is termed as a compression member. Compression members in buildings are called as columns and those found in trusses are called struts or rafters. Although the structural column is a simple structural member, the interaction between the responses and characteristics of the material, the cross section, the method of fabrication, the imperfections and other geometric factors, and end conditions (restraints) make the column one of the most complex individual structural members.

The following parameters have an effect on the column strength:

1. Material properties (stress–strain relationship, yield stress)
2. Manufacturing method (hot-rolled/welded and cold-straightened shape resulting in different residual stress patterns)
3. Shape of the cross section (area, cross section geometry, and bending axis)
4. Length
5. Initial out-of-straightness (maximum value and shape of the distribution) and other imperfections (see also Tables 33 and 34 of IS 800: 2007)
6. End support conditions (pinned-with or without sway and restrained ends, with or without sway)

All these parameters that affect the strength of columns are discussed. The possible failure modes of columns are identified. The behaviour of compression members differ based on their length. Short or stocky columns can be loaded up to their yield stress and can attain their squash loads, provided the elements that make up the cross section are prevented from local buckling. Long compression members buckle elastically and hence their strength may be predicted by the Euler's formula. Intermediate length compression members fail both by yielding and buckling and hence their behaviour is inelastic. The behaviour of these slender columns is affected by the various parameters as listed before. To understand their behaviour, the elastic buckling of ideal pin-ended column is considered and Euler formula is derived. Then the effects of initial out-of-straightness, residual stresses, and material properties on the strength of compression members are discussed. Since there is a wide scatter in the strength of columns with different cross sections, many international codes have gone in for multiple column curves. Multiple column curves were first suggested by ECCS, which were followed by the American SSRC column strength curves. The Indian code has adopted the column curves similar to those given in the British code. The various individual and built-up or combined cross sections which can be used as compression members are discussed.

The effect of end supports (end restraints) on column strength is usually incorporated in the design by the concept of effective length. The effective length of columns with idealized boundary conditions are discussed and also the effect of intermediate restraints and the calculation of effective lengths of compressive members in different planes. The effective length of columns in multi-storey buildings is difficult to calculate, since the end restraints depend on the stiffness of the beams meeting at column ends and also on whether the frames are braced or unbraced. Columns in sway frames buckle in double curvature (hence the effective length factor will always be greater than unity) and the columns in braced (non-sway) frames, buckle in single curvature (hence the effective length factor will be less than unity).

The two widely used charts (alignment chart developed by Julian and Lawrence and Wood's curves) for the determination of effective length factor of columns in both braced and unbraced multi-storey buildings are illustrated. Since these charts are based on some idealized conditions, some modifications have been proposed by various researchers. The IS code provisions on effective length are based on the Wood's curves, with slight modifications.

The compression members in trusses have been considered separately since the loads are usually applied at the joints and the joints are often considered as pinned. Similarly, single-angle struts should also be treated separately, since they will be connected in most of the cases through one of their legs to the other members. The IS code provisions for such continuous and discontinuous angle members are summarized. Discussions on struts with variable cross section (tapered columns) and stepped columns are also included.

Columns for which the shear centre and centroid do not coincide will buckle in a coupled flexural and torsional mode. Sometimes we may also encounter columns with no axis of symmetry. Hence, a brief discussion about these columns and methods to prevent buckling failures is included. IS code provisions for the design of built-up columns with lacings and battens are also provided. The various steps involved in the design of compression members are also given. Some specifications for the compression members composed of two components back-to-back are also included. A brief description of compression members with other material properties and shapes, including steel arches is provided. Most of the concepts are explained with illustrative examples.

MULTIPLE-CHOICE QUESTIONS

9.1 A vertical member primarily subjected to compression is called
(a) principal rafter (c) beam
(b) strut (d) stanchion

9.2 The failure mode in which an axially loaded compression member may fail is by
(a) local buckling (c) squashing
(b) overall flexural buckling (d) all of these

9.3 Which of the following sections are preferred as columns?
(a) ISLB (b) ISMB (c) ISHB (d) ISWB

9.4 It is a good practice to use the following section as a compression member.
(a) Plastic or compact section
(b) Semi-compact section
(c) Slender section
(d) Compact or semi-compact section

9.25 The thickness of the lacing bar for single lacing should not be less than __________
(a) 1/30th of effective length
(b) 1/40th of effective length
(c) 1/50th of effective length
(d) 1/60th of effective length

9.26 For connecting lacing bars to column sections with 16 mm bolts, the width of lacing bar should not be less than __________
(a) 40 mm (b) 45 mm (c) 48 mm (d) 35 mm

9.27 The effective length of a battened column is __________
(a) increased by 10%
(b) increased by 5%
(c) 0.8 times the actual length of column
(d) 0.9 times the actual length of column

9.28 The effective depth of end batten should be more than __________
(a) twice the flange width of component column
(b) perpendicular distance between the centroids of main members
(c) longitudinal distance between innermost fasteners
(d) both (a) and (b)

9.29 The maximum lateral deflection allowed in a compression member in industrial buildings with elastic cladding, as per IS:800 is __________
(a) *L*/250 (b) *L*/325 (c) *L*/150 (d) *L*/240

9.30 The maximum lateral deflection allowed in a compression member in industrial buildings, supporting cab operated gantry girder is __________
(a) *L*/150 (b) *L*/100 (c) *L*/240 (d) *L*/360

EXERCISES

Note: Use grade Fe 410 steel (f_y = 250 MPa) and $E = 2.0 \times 10^5$ MPa, if not specified in the problem.

9.1 Check whether an ISHB 400 in Fe 540 steel would be affected by local buckling effects when used as a column.

9.2 Determine the axial load capacity of a short length of square box column in Fe 540 steel fabricated by welding together four 600 × 14 mm plates.

9.3 A column consisting of ISHB 400 at 774 N/m has an unsupported length of 3.8 m. It is effectively held in position at both ends, restrained against rotation at one end. Calculate the axial load this column can carry.

9.4 Determine the design axial load of a column section ISHB 300. The column is having a height of 9 m and is effectively restrained by two bracings in the *y*-*y* direction at 3 m and 6 m and by one bracing member in the *z*-*z* direction at mid-height. Assume pinned condition at both ends of the column.

9.5 Determine the capacity of a column of size (wide flange section) W 250 × 250 × 101 (with weight = 1010 N/m, r_z = 112.7 mm, r_y = 65.6 mm, area = 12900 mm^2) with bracing and end condition as shown in Fig. 9.66.

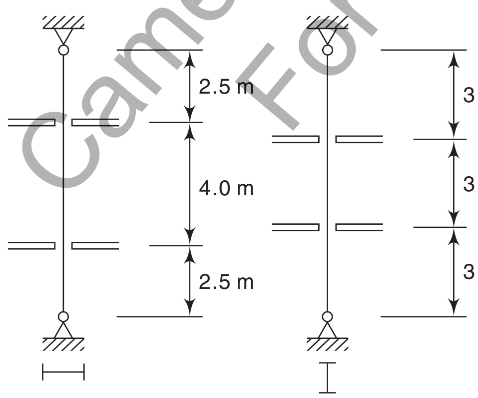


FIG. 9.66

9.6 For the same bracing and end condition as shown in Fig. 9.66 determine the capacity of ISMB 550 (weight = 1040 N/m) and compare the capacity with the W250 × 250 × 101 section of the previous example. (*Note:* Both the sections are having more or less equal weight.)

9.7 Determine the axial load capacity of the following section, assuming that the column has pinned ends and has a length of 3 m.
(a) ISMB 400 with one cover plate of size 250 × 12 mm on each flange [see Fig. 9.24(n)]
(b) ISMB 300 with one channel ISMC 300 welded to each flange of ISMB 300 [see Fig. 9.24(q)]
(c) Two ISMC 300 back-to-back with a distance of 200 mm between them [see Fig. 9.24(k)]
(d) Four angles ISA 90 × 90 × 6 with a plate 300 × 12 mm welded to form a column as shown in Fig. 9.24(r).

9.8 A single angle discontinuous strut ISA 65 × 65 × 6 is 1.2-m long. It is connected by one bolt at each end. Calculate the safe load this strut can carry.

9.9 A double angle discontinuous strut consists of two ISA 75 × 75 × 6 connected back-to-back to both sides of the 10-mm thick gusset plate with two bolts. The length of the strut is 4.5 m. Calculate the safe load carrying capacity of the section.

9.10 A double angle discontinuous strut 2.25 m long consists of two ISA 60 × 60 × 5 connected to the same side of the 8-mm thick gusset plate by one bolt. Calculate the load that this strut can carry.

9.11 An ISHB 400 column is an interior column with strong axis buckling in the plane of the frame as shown in Fig. 9.67. The columns above and below are also ISHB 400. The beams framing at the top are ISMB 400 and those at the bottom are ISMB 450. The columns are 4-m high and the beam span is 7 m. Determine the effective length of this column (*AB* in Fig. 9.67) as per the alignment chart as well as Wood's curves of IS 800 : 2007. Assuming that the column carries a factored load of 1200 kN, determine whether the column is safe to carry this load. Assume that the frame is braced by shear walls.

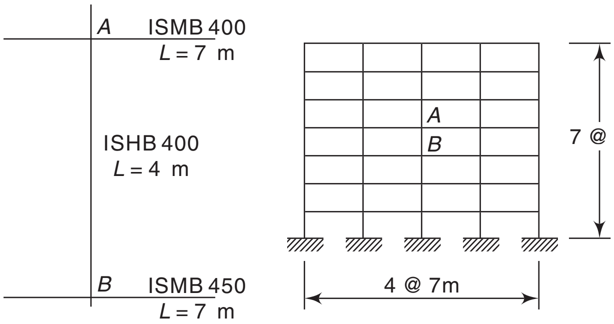


FIG. 9.67

9.12 For the same building shown in Fig. 9.67, determine the effective length and load carrying capacity of column *AB*, if the frame is a sway frame. Compare the results with those of Exercise 9.11.

9.13 A part of an office building and the elevator of the internal column stack *A* are similar to those shown in Fig. 9.67, except that the distance between the columns in *X*- and *Y*-direction is 6 m. Assuming the following loads, design an interior ground floor column for axial loads only.
Roof: Dead load (total) = 3 kN/m^2
Imposed load = 1.5 kN/m^2
Floors: Dead load (total) = 4 kN/m^2
Imposed load = 3 kN/m^2
Take the self-weight of the column including the fire protection as 0.8 kN/m. Also assume that the roof and floor have the same layout.

9.14 Design a column to support an axial load of 2500 kN, assuming that the column has an effective length of 8 m with respect to the *z*-axis and 4 m with respect to the *y*-axis. Since the maximum available rolled section (ISHB 450) may not be sufficient to support this load, design a built-up I-section with two flange plates.

9.15 Design a hollow circular tubular strut of a truss having an effective length of 1.8 m and to carry a factored load of 150 kN.

9.16 For the problem in Exercise 9.15, design a suitable hollow rectangular section and compare the weights.

9.17 Design a suitable section for a column 3.85-m long which is effectively held in position and restrained against rotation at both ends, in order to carry a factored load of 450 kN.

9.18 Design a single angle strut carrying a factored compressive load of 88 kN with length between centre-to-centre of intersection as 2.2 m. Design also the bolted end connections.

9.19 Design a single angle section having 3.25 m length to carry a factored compressive load of 14.5 kN due to wind. Design also the welded end connections for it.

9.20 Design a double angle section to act as a compression member in a truss having 2.1 m length and to carry a factored load of 150 kN.
(a) Connected back-to-back to both sides of a 12-mm thick gusset plate.
(b) Connected back-to-back to one side of the 12-mm thick gusset.
(c) Angles in star formation

9.21 Design a built-up column with two channels back-to-back having an effective length of 6.5 m and carrying a factored load of 1000 kN. Also design the lacings.

9.22 Design a built-up column with four angles laced together, having an effective length of 7.0 m and supporting a factored load of 1200 kN.

9.23 Design a built-up column with two channels toe-to-toe to carry a factored load of 2000 kN. Take the effective length as 5.5 m.
(a) Design it as a laced column and also design the lacings.
(b) Design it as a battened column and also design the battens.

9.24 Design a column of length 8 m to carry a factored axial load of 3000 kN, the ends having fixed in both position and direction. Design a suitable built-up column section using I-sections and batten plates.

9.25. Design a circular stainless steel column to carry a factored load of 200 kN and having an effective length of 5 m. Assume S 316 grade stainless steel.

9.26 A steel tower supports a water tank of size 3 m × 3 m × 2.5 m. The self-weight of the tank is 45 kN, when the tank is empty. The arrangement of the structure is shown in Fig. 9.68. Take unit weight of water as 9.81 kN/m^3. Take design wind pressure as per IS 875-Part 3, assuming that the tank is situated in New Delhi. Design the tower structure for the water tank.

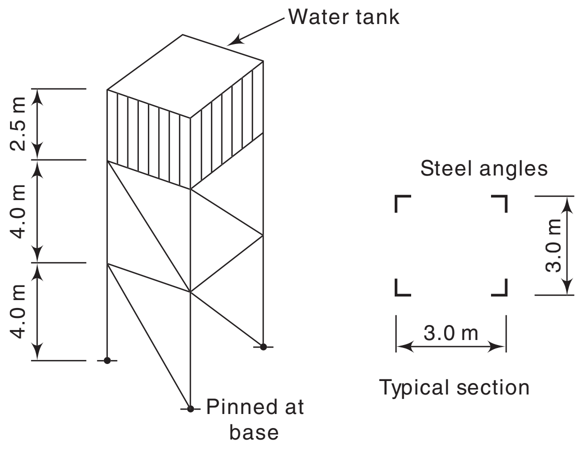


FIG. 9.68

REVIEW QUESTIONS

9.1 Distinguish between column and beam column.

9.2 Cite the instances when a column may be regarded as an axially loaded column.

9.3 What is the basic difference in behaviour between tension and compression members, while resisting the loads?

9.4 Can bolt holes be ignored in the design of compression member? If yes, why?

9.5 State the parameters that affect the strength of compression members.

9.6 State the possible failure modes of an axially loaded column.

REFERENCES

Adluri S.M.R. and Madugula, M.K.S., 'Development of Column Curves for Single Angles', *Journal of Structural Engineering*, Vol. 122, No. 3, March 1996, pp. 318–325.

Agarval, K.M. and Stafiej, A.P., 'Calculation of Effective Length of Stepped Columns', *Engineering Journal*, AISC, 1980, No. 4, pp. 96–105.

ANSI/AISC 360-10, *Specification for Structural Steel Buildings*, American Institute of Steel Construction, Chicago, IL, 2010 (www.aisc.org).

AISC, *Specification for Load and Resistance Factor Design of Single-angle Members*, American Institute of Steel Construction, Chicago, USA, Dec 1993.

Allen, D., 'Merchant–Rankine Approach to Member Stability', *Journal of Structural Div.*, ASCE, Vol. 104, No. ST12, Dec 1978, pp. 1904–1914.

Allen, H.G. and Bulson, P.S., *Background to Buckling*, McGraw-Hill Book Company (UK) Ltd, London, 1980.

Anderson, J.P. and Woodward, J.H., 'Calculation of Effective Lengths and Effective Slenderness Ratios of Stepped Columns', *Engineering Journal*, AISC, 1972, No. 4, pp. 157–166.

ANSI/ASCE, 10-90, *Design of Latticed Steel Transmission Structures*, ASCE, New York, 1992.

ANSI-ASCE Standard No. ASCE-8-90, *Specification for the Design of Cold-formed Stainless Steel Structural Members*, ASCE, New York, 1991.

Arya, A.S. and Ajmani, J.L., *Design of Steel Structures*, Fourth edition, Nem Chand & Bros, Roorkee, 1989.

ASCE Task Committee on Effective Length—Effective Length and Notional Load Approaches for Assessing Frame Stability: Implications for American Steel Design, American Society of Civil Engineers, New York, 1997.

Aslani, F. and S.C. Goel, 'An Analytical Criterion for Buckling Strength of Built-up Compression Members', *Engineering Journal*, AISC, Vol. 28, No. 4, Fourth quarter 1991, pp. 159–168.

Ayrton, W.E. and Perry, J., 'On Struts', *The Engineer*, Vol. 62, 1866, p. 464.

Ballio, G. and Mazzolani, F.M., *Theory and Design of Steel Structures*, Chapman and Hall, London, 1983, p. 632.

Ballio, G. and Mazzolani, F.M., *Theory and Design of Steel Structures*, Chapman and Hall, London, 1983, pp. 505–506.

Bathon, L., Mueller III, W.H., and Kempner, Jr., L., 'Ultimate Load Capacity of Single Steel Angles', *Journal of Structural Engi*neering, ASCE, Vol. 119, No. 1, Jan 1993, pp. 279–300.

Beer, H. and Schultz, G., 'Theoretical Basis of the European Column Curves', *Construction Metallique*, No. 3, Sep 1970, pp. 37–57.

Bjorhovde, R. and Tall, L., 'The Probabilistic Characteristics of Maximum Column Strength', *Proc. of the International Colloquium on Column Strength*, Paris, 23–34 Nov 1971.

Bjorhovde, R., 'Deterministic and Probabilistic Approaches to the Strength of Steel Columns', PhD Dissertation, Lehigh Univ., Bethlehem, PA, May 1972.

Bjorhovde, R., 'Effect of End Restraint on Column Strength—Practical Applications', *Engineering Journal*, AISC, Vol. 21, No. 1, 1985, pp. 42–45.

Bleich, F., *Buckling Strength of Metal Structures*, McGraw-Hill, New York, 1952.

Bradford, M.A. and Pi Y-L., 'Flexural–Torsional Buckling of Fixed Arches under Uniform Bending', *Journal of Constructional Steel Research*, Vol. 62, No. 1–2, Jan–Feb 2006, pp. 20–26.

Bridge, R.Q. and Fraser, D.J., 'Improved G-factor Method for Evaluating Effective Lengths of Columns', *Journal of Structural Engineering*, ASCE, Vol. 113, No. 6, Jun 1987, pp. 1341–1356.

BS 5950-1: 2000 *Structural Use of Steelwork in Building – Part 1: Code of Practice for Design—Rolled and Welded Sections*, British Standard Institution, London, Aug 2001.

Chen, W.F. and Lui, E.M., *Stability Design of Steel Frames*, CRC Press, Boca Raton, Florida, 1991.

Chen, W.F. and Lui, E.M., *Structural Stability—Theory and Implementation*, Elsevier, New York, 1987.

Cheong – Siat – Moy, F., 'An Improved K-factor Formula', *Journal of Structural Engineering*, Vol. 125, No. 2, Feb 1999, pp. 169–174.

Christopher, J.E. and Bjorhovde, R., 'Semi-rigid Frame Design Methods for Practicing Engineers', *Engineering Journal*, AISC, Vol. 36, First quarter 1999, pp. 12–28.

Chu, K.H. and Chow, H.L., Effective Column Length in Unsymmetrical Frames, *Publication of IABSE*, 1969, pp. 29–31.

Dafedar, J.B., Desai, Y.M., and Shiyekar, M.R., 'Review of Code Provisions for Effective Length of Framed Columns', *The Indian Concrete Journal*, Vol. 75, No. 6, Jun 2001, pp. 402–407.

DIN 18800, Teil 2, *Stahlbauten: Stabilitätsfälle, Knicken von Stäben und Stabwerben,* Deutsches Institut fur Normung e.v., Berlin, Dec 1980.

Disque, R.O., 'Inelastic K-factor for Column Design', *Engineering Journal*, AISC, Vol. 10, No. 2, Second quarter 1973, pp. 33–35.

Dowling P.J., Knowles, P., and Owens, G., (ed.) *Structural Steel Design*, Butterworths, London and The Steel Construction Institute, Ascot, 1988.

Driscoll, G.C., 'Effective Length of Columns with Semi-rigid Connections', *Engineering Journal*, AISC, Vol. 13, No. 4, Fourth quarter 1976, pp. 109–115.

Duan, L. and Chen, W.F., 'Effective Length Factor Columns in Unbraced Frames', *Journal of Structural Engineering*, ASCE, Vol. 115, No. 1, Jan 1989, pp. 149–165.

Duan, L. and Chen, W.F., Effective Length Factor for Columns in Braced Frames, *Journal of Structural Engineering*, ASCE, Vol. 114, No. 10, Oct 1988, pp. 2357–2370.

Engesser, F., Über die knickfestigkeit gerader stäbe, *Zeitschrift des Architekten-und Ingenieur-Vereins zu Hannover*, Vol. 35, 1889, pp. 455–462.

Epstein, H.I. and Thacker, B., "Effect of bolt stagger for block shear tension failures in angles", *Computers and Structures*, Vol.39, No.5, 1991, pp.571-76.

Epstein, H.I. and Aleksiewicz , L.J., 'Block Shear Equations Revisited Again', *Engineering Journal*, AISC, Vol. 45, No. 1, First quarter 2008, pp. 5–12.

Euler, L., Sur le Force de Colonnes, *Memoires de l'Academie Royale des Sciences et Belles, Lettres*, Vol. 13, Berlin, 1959 (English translation by J.A.Van den Broek, *American Journal of Physics*, Vol. 15, 1947, pp. 309).