1

Numbers

CHAPTER

Learning Objective

In this chapter, you will:

- Understand the concept of number system with different types of numbers and their classification
- Understand the hierarchy of arithmetic operations – BODMAS rule
- Learn about various divisibility rules and their applications
- You will understand all about factors of a number including:
  - Finding the number of factors
  - Number of ways of expressing a given number as a product of two factors
  - Sum of all the factors of a number
  - Product of all the factors of a number
  - Number of ways of writing a number as product of two co-primes
  - Number of co-primes to N, that are less than N
  - Sum of co-primes to N, that Are Less than N
- Understand different methods to calculate H.C.F, L.C.M of some numbers such as:
  - Factorization method
  - Long Division method for H.C.F
- Learn about the L.C.M and H.C.F models
- Master successive division
- Acquire skills to find the index of the greatest power of a number in N!
- Understand how to calculate the last digit and the last two digits using cyclicity/pattern method
- Learn about Remainder Theorem and its applications
- Familiarize yourself with useful theorems to calculate remainders of complex expressions such as
  - Binomial Theorem
  - Fermat's Little Theorem
  - Wilson's Theorem

NUMBERS is one of the most important topics required for competitive entrance exams. In this chapter, we have put together a number of models of problems—mainly based on various problems that have been appearing in different exams.

❐ BASIC ARITHMETIC OPERATIONS

Addition is the most basic operation. We have an intuitive understanding of the operation. It is the process of finding out the single number or fraction equal to two or more quantities taken together. The two (or more) numbers that are added are called addends and the result of the addition is called the sum. **For two numbers *A* and *B*, this is denoted as *A* + *B*.**

Subtraction is the process of finding out the quantity left when a smaller quantity (number or fraction) is reduced from a larger one. This is called the difference of the two numbers. The word difference is taken to mean a positive quantity, i.e., the difference of 10 and 8 is 2. The difference of 8 and 10 is also 2. This is also referred to as the remainder.

Multiplication is repeated addition. The number that is added repeatedly is the multiplicand. The number of times it is added is the multiplier. The sum obtained is the product.

For example, in the multiplication $3 \times 4 = 12$, 3 is the multiplicand, 4 is the multiplier and 12 is the product.

Division is repeated subtraction. From a given number, we subtract another repeatedly until the remainder is less than the number that we are subtracting. The number from which we are subtracting the second one is the dividend. The number that is subtracted repeatedly (the second one) is the divisor. The number of times it is subtracted is the quotient. The number that remains after we are done subtracting is the remainder. Division can also be thought of as the inverse of multiplication. ***A*/*B* is that number with which B has to be multiplied to get *A*.**

For example, in the division 32/5, 32 is the dividend, 5 is the divisor, 6 is the quotient and 2 is the remainder.

Involution (or raising to the power *n*) is repeated multiplication. Therefore, a^n is the the product of *n* *a*'s. Here, *a* is the base, *n* is the index and a^n is the *n*th power of *a*.

For example, $a \times a = a^2$, which is the second power of *a* and $a \times a \times a = a^3$, which is the third power of *a*.

Evolution is the inverse of involution. The *n*th root of a number is that number whose *n*th power is the given number. The root of any number or expression is that quantity which when multiplied by itself the requisite number of times produces the given expression.

For example, the square root of *a*, $\sqrt{a}$ when multiplied by itself two times, gives a; similarly, the cube root of *a*, $\sqrt[3]{a}$ when multiplied by itself three times, gives *a*.

All the above operations are performed in Algebra also. Algebra treats quantities just as Arithmetic does, but with greater generality, for algebraic quantities are denoted by symbols which may take any value we choose to assign them as compared to definite values usually used in arithmetic operations.

Rule of Signs

The product of two terms with like signs is positive; the product of two terms with unlike signs is negative.

Example: $-1 \times -1 = +1$; $+1 \times -1 = -1$; $+1 \times +1 = +1$; $-1 \times +1 = -1$

CLASSIFICATION OF REAL NUMBERS

Real Numbers are classified into rational and irrational numbers.

Rational Numbers

A number which can be expressed in the form *p*/*q* where *p* and *q* are integers and $q \neq 0$ is called a rational number.

For example, 4 is a rational number since 4 can be written as 4/1 where 4 and 1 are integers and the denominator $1 \neq 0$. Similarly, the numbers 3/4, –2/5, etc. are also rational numbers.

Recurring decimals are also rational numbers. **A recurring decimal is a number in which one or more digits at the end of a number after the decimal point repeats endlessly** (For example, 0.333..., 0.111111..., 0.166666..., etc. are all recurring decimals). Any recurring decimal can be expressed as a fraction of the form *p*/*q*, and hence, it is a rational number. We will study in another section in this chapter the way to convert recurring decimals into fractions.

Between any two numbers, there can be infinite number of other rational numbers.

Irrational Numbers

Numbers which are not rational but which can be represented by points on the number line are called irrational numbers. Examples for irrational numbers are $\sqrt{2}, \sqrt{3}, \sqrt[4]{5}, \sqrt[3]{9}$, etc.

Numbers like *π*, *e* are also irrational numbers.

Between any two numbers, there are infinite numbers of irrational numbers.

Another way of looking at rational and irrational numbers is as follows:

Terminating decimals and recurring decimals are both rational numbers.

Any non-terminating, non-recurring decimal is an irrational number.

Integers

All integers are rational numbers. Integers are classified into negative integers, zero and positive integers. Positive integers can be classified as Prime Numbers and Composite Numbers. In problems on Numbers, we very often use the word 'number' to mean an 'integer.'

Prime Numbers

A number other than 1 which does not have any factor apart from one and itself is called a prime number.

Examples for prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, etc.

There is no general formula that can give prime numbers. Every prime number greater than 3 can be written in the form of $(6\boldsymbol{k} + 1)$ or $(6\boldsymbol{k} - 1)$ where ***k*** is an integer. For the proof of this, refer to 4th point under 'Some important points to note' given later on in this chapter.

Composite Numbers

Any number other than 1, which is not a prime number is called a composite number. In other words, a composite number is a number which has factors other than one and itself.

Examples for composite numbers are 4, 6, 8, 9, 10, 14, 15, etc.

NOTE

The number 1 is neither prime nor composite.
The only prime number that is even is 2.
There are 15 prime numbers between 1 and 50 and 10 prime numbers between 50 and 100. So, there are a total of 25 prime numbers between 1 and 100.

❐ Even and Odd Numbers

Numbers divisible by 2 are called even numbers whereas numbers that are not divisible by 2 are called odd numbers.

Examples for even numbers are 2, 4, 6, 8, 10, etc. Examples for odd numbers are 1, 3, 5, 7, 9, etc.

NOTE

- Every even number ends in 0, 2, 4, 6 or 8.
- The sum of any number of even numbers is always even.
- The sum of odd number of odd numbers (i.e., the sum of 3 odd numbers, the sum of 5 odd numbers, etc.) is always odd whereas the sum of even number of odd numbers (i.e., the sum of 2 odd numbers, the sum of 4 odd numbers, etc.) is always even.
- The product of any number of odd numbers is always odd.
- The product of any number of numbers where there is at least one even number is even.

❐ Perfect Numbers

A number is said to be a perfect number if the sum of ALL its factors excluding itself (but including 1) is equal to the number itself.

For example, 6 is a perfect number because the factors of 6, i.e., 1, 2 and 3 add up to the number 6 itself.

Other examples of perfect numbers are 28, 496, 8128, etc.

❐ HIERARCHY OF ARITHMETIC OPERATIONS

To simplify arithmetic expressions, which involve various operations like brackets, multiplication, addition, etc. a particular sequence of the operations has to be followed. For example, 2 + 3 × 4 has to be calculated by multiplying 3 with 4 and the result 12 added to 2 to give the final result of 14 (you should not add 2 to 3 first to take the result 5 and multiply this 5 by 4 to give the final result as 20). This is because in arithmetic operations, multiplication should be done first before addition is taken up.

The hierarchy of arithmetic operations are given by a rule called BODMAS rule. The operations have to be carried out in the order in which they appear in the word BODMAS, where different letters of the word BODMAS stand for the following operations:

FORMULA

B	Brackets
O	Of
D	Division
M	Multiplication
A	Addition
S	Subtraction

There are four types of brackets:

(i) **Vinculum:** This is represented by a bar on the top of the numbers. For example,

$2 + 3 - \overline{4 + 3}$; Here, the figures under the vinculum have to be calculated as 4 + 3 first and the 'minus' sign before 4 is applicable to 7. Therefore, the given expression is equal to 2 + 3 − 7 which is equal to −2.

(ii) **Simple brackets:** These are represented by ( )

(ii) **Curly brackets:** These are represented by { }

(iv) **Square brackets:** These are represented by [ ]

The brackets in an expression have to be opened in the order of vinculum, simple brackets, curly brackets and square brackets, i.e., [{ (¯) }] to be opened from inside outwards.

After brackets is O in the BODMAS rule standing for 'of' which means multiplication. For example, 1/2 of 4 will be equal to 1/2 × 4 which is equal to 2.

After O, the next operation is D standing for division. This is followed by M standing for multiplication. After Multiplication, A standing for addition will be performed. Then, *S* standing for subtraction is performed.

Two operations that have not been mentioned in the BODMAS rule are taking powers and extracting roots, viz, involution and evolution, respectively. When these operations are also involved in expressions, there is never any doubt about the order in which the steps of the simplification should be taken. The sign for root extraction is a variant of the vinculum and for powers, brackets are used to resolve ambiguities in the order.

❐ RECURRING DECIMALS

A decimal in which a digit or a set of digits is repeated continuously is called a recurring decimal. Recurring decimals are written in a shortened form, the digits which are repeated being marked by dots placed over the first and the last of them, therefore,

$$\frac{8}{3} = 2.666..... = 2.\dot{6} \text{ or } 2.\overline{6};$$

$$\frac{1}{7} = 0.142857142857142857... = 0.\overline{142857}$$

Exercise-2

1. 38
2. (A)
3. (i) 1
   (ii) 7 or 19
   (iii) 1 or 13 or 17
4. 96
5. 1
6. (i) 36
   (ii) 75
   (iii) 36
7. (i) 57
   (ii) 07
8. 27
9. (i) 3, 4
   (ii) 6, 8
   (iii) 6, 32
10. 480
11. 0
12. 600
13. 4
14. (D)
15. 1.7
16. 6
17. (A)
18. (i) 2
    (ii) 1
19. (D)
20. 120
21. (C)
22. 131
23. 01
24. 1
25. 92
26. (C)
27. 2
28. (C)
29. 180
30. (D)
31. 1
32. 3
33. (A)
34. 30
35. 616
36. (C)
37. (i) 30
    (ii) 900
38. (A)
39. 9
40. 7
41. (A)
42. (D)
43. 50
44. (A)
45. 210
46. (C)
47. 18, 648
48. (D)
49. 43
50. (B)
51. 19
52. 10
53. 8
54. 55
55. (i) 15
    (ii) 5
56. 5
57. 0, 0
58. 28
59. 10
60. 72
61. (i) 6, 16
    (ii) 4, 8
62. 1
63. 38
64. (i) 10, 36
    (ii) 186, 7644
    (iii) 80^5, 2340^{18}
65. (D)
66. 32
67. 5, 7, 8
68. 13
69. 120, 1, 315
70. (A)
71. (D)
72. (D)
73. (C)
74. (D)
75. (C)
76. (D)
77. (D)
78. (B)
79. (A)
80. (D)
81. (B)
82. 77
83. (D)
84. (C)
85. (C)
86. (D)
87. (B)
88. (A)
89. 40
90. 3
91. (A)
92. (C)
93. (B)
94. 10
95. (D)
96. (A)
97. 2
98. 8
99. (B)
100. 563, 495

Exercise-3

1. (A)
2. 60
3. 4
4. (D)
5. 6
6. (C)
7. (D)
8. (A)
9. (D)
10. 26
11. (B)
12. (C)
13. (D)
14. (B)
15. 3050
16. (D)
17. (C)
18. (A)
19. (B)
20. (D)
21. 483840
22. 18
23. (D)
24. (D)
25. (A)
26. 24
27. 87
28. 5
29. (B)
30. (D)
31. (D)
32. (B)
33. (D)
34. (B)
35. 255
36. 21690
37. (D)
38. 1
39. (C)
40. (B)
41. (A)
42. (C)
43. (C)
44. 397
45. (C)
46. (B)
47. 2
48. 4.5
49. (C)
50. (C)
51. (B)
52. 2
53. 4
54. 87
55. (C)
56. 29
57. (B)
58. 1
59. 5
60. (B)
61. (A)
62. 3600
63. 1, 40
64. 666
65. (A)
66. (B)
67. 16
68. 1053, 150
69. (B)
70. 16
71. 12
72. 9
73. 37
74. 2
75. (C)
76. 22122
77. 8
78. (D)
79. (C)
80. 25
81. 8
82. 2
83. 26
84. 181
85. 0
86. (B)
87. 77 or 149
88. 200
89. 37
90. (A)
91. 459
92. (C)
93. 249
94. 5
95. 3
96. 88 or 40
97. 837
98. 131
99. (D)
100. 19

Exercise-4

1. (D)
2. (B)
3. (C)
4. 6
5. 1249 9488
6. (C)
7. (B)
8. (B)
9. (B)
10. 0
11. 3
12. (C)
13. (D)
14. (B)
15. 5
16. 278
17. 15
18. (A)
19. 30
20. (B)
21. 24
22. (C)
23. (A)
24. (A)
25. (C)
26. 0
27. (C)
28. (A)
29. 1
30. (C)
31. 2732
32. 663936
33. (B)
34. 7
35. (C)
36. (D)
37. (B)
38. 26
39. (D)
40. (C)
41. 1296
42. (A)
43. (A)
44. (D)
45. 33
46. (C)
47. (B)
48. 1872
49. (B)
50. 2

Exercise-5

1. (C)
2. (C)
3. (B)
4. (D)
5. (C)
6. (C)
7. (A)
8. (B)
9. (B)
10. (A)
11. (A)
12. (A)
13. (A)
14. (A)
15. (A)
16. (D)
17. (C)
18. (D)
19. (B)
20. (A)
21. (A)
22. (D)
23. (B)
24. (C)
25. (D)
26. (A)
27. (D)
28. (C)
29. (C)
30. (D)
31. (C)
32. (A)
33. (A)
34. (C)
35. (C)
36. (A)
37. (C)
38. (C)
39. (C)
40. (A)
41. (B)
42. (C)
43. (C)
44. (B)
45. (A)

SOLUTIONS

EXERCISE-1

1. $x = 56^y + 1 = 56^y + 1^y$ which is divisible by 57 only when y is odd ($\because$ $a^N + b^N$ is divisible by $a + b$ only when N is odd). $\therefore$ It is divisible by any factor of 57 only when y is odd.

$\therefore$ It is divisible by 19 when y is odd.

2. The number of digits in the product must be at least the number of digits in (10^6) (10^9) (10^{11}) and less than the number of digits in (10^7) (10^{10}) (10^{12}).

$\therefore$ The number has at least 27 digits and less than 30 digits.

3. The L.C.M. of the given expression is $2^3 \times 3^3 \times 5^2$.

4. The product of two or more even numbers is always even.

5. Let the greatest number be N.

Let the remainder be r. $93 - r$, $131 - r$ and $188 - r$ must be divisible by N

$\therefore$ $131 - r - (93 - r)$ and $188 - r - (131 - r)$ must be divisible by N.

$\therefore$ 38 and 57 must be divisible by N.

N is the greatest possible number satisfying this condition

$\therefore$ N = H.C.F. (38, 57) = 19.

6. $38^{2n} - 11^{2n} = (38^2)^n - (11^2)^n = (1444)^n - (121)^n$. This is always divisible by $1444 - 121 = 1323$. The greatest number which divides it among the choices is 1323.

7. The sum of the alternate digits starting from the units digit $= 8 + 3 + 2 + 5 + 6 + 9 = 33$.

The sum of the alternate digits starting from the tens digit is $7 + 1 + 7 + y + 8 + 1 = 24 + y$. The 11's remainder of the number is equal to the 11's remainder of $33 - (24 + y) = 9 - y$. This is 0 as the number is divisible by 11.

$\therefore$ $y = 9$.

8. $3^{200} = 3^4 \times ^{(50)}$ As the index of the power of 3 is divisible by 4, 3^{200} has the same units' digit as 3^4, i.e., 1.

4^{500} has an even index.

Its units' digit is 6.

$\therefore$ Units' digit of (3^{200}) (4^{500}) is 6.

9. The smallest four-digit number is 1000. If 1000 is divided by 112, the remainder is 104.

$112 - 104 = 8$. If 8 is added to 1000, the resulting number will be the smallest four-digit number that is a multiple of 112.

10. Let the number be N. Let the quotient obtained when the number was divided by 32 be q.

$N = 32q + 29$

When q is of the form $2k + 1$ where k is any whole number, then $N = 64\ k + 32 + 29 = 64k + 61$. When $64k + 61$ is divided by 64 the remainder is 61. When q is of the form $2k$ where k is a whole number then $N = 64k + 29$. When $64k + 29$ is divided by 64 the remainder is 29.

$\therefore$ The remainder is either 61 or 29.

11. Suppose a number x has m digits, i.e., $10^{m-1} \le x < 10^m$

$\therefore$ $10^{2m-2} \le x^2 < 10^{2m}$, i.e., x^2 has $2m$ or $2m - 1$ digits. Conversely, if a number has $2m - 1$ or $2m$ digits, its square root has m digits. Therefore, if a number has 13 digits, its square root has 7 digits.

12. Sum of the digits of 7654321A = 28 + A, so it must be divisible by 9. As $0 \le A \le 9$, $28 \le 28 + A \le 37$. Only when $28 + A = 36$ is the number divisible by 9.

$\therefore$ A = 8.

13. Divisors = 5, 6, 7, 8, 9

Remainders = 4, 5, 6, 7, 8.

The difference between the respective divisors and remainders is constant.

The required number = (L.C.M. of divisors) – Common difference = 2520 – 1 = 2519

14. (H.C.F.) (L.C.M.) = Product

$\therefore$ 18 (L.C.M.) = 3240

$\therefore$ L.C.M. = 180

15. 63 and 91 are numbers that leave a remainder of 7, when divided by 28. When divided by 35, they leave remainders of 28 and 21, respectively.

16. $(2PQR)^4$ must be at least $(2000)^4$ and less than $(3000)^4$. $(2000)^4$ as well as $(3000)^4$ have 14 digits.

$\therefore$ $(2PQR)^4$ also has 14 digits.

17. D is odd.

$\therefore$ $41^D + 7^D$ must be divisible by $41 + 7 = 48$

$\therefore$ It is divisible by any factor of 48. Only is not a factor of 48.

18. Let the other number be x.

(L.C.M.) (H.C.F.) = product of the numbers

$(264)\ (2) = (22)\ (x) \Rightarrow x = 24$

19. Let the numbers be a and b where $a \leq b$. We consider the possibilities $a < b$ and $a = b$.

Possibility 1: $a < b$ (1)

If b is divisible by a, L.C.M. = b and H.C.F. = a.

Otherwise L.C.M. > b and H.C.F. < a

∴ L.C.M. ≥ b and H.C.F. ≤ a

(1) ⇒ L.C.M. ≥ $b > a \geq$ H.C.F.

L.C.M. > H.C.F.

Possibility 2: $a = b$

L.C.M. = H.C.F. = each number

The numbers must be equal for H.C.F. = L.C.M. to hold true.

20. Units digit of $(13687)^{3265}$ is the same as units digit of $7^{3265} = 7^{4\,(816)+1}$

∴ Units digit of 7^{3265} is the same as that of 7^1, i.e., 7.

21. $$\text{L.C.M.}\left(\frac{5}{6},\frac{9}{10},\frac{8}{9}\right) = \frac{\text{L.C.M.}(5,9,8)}{\text{H.C.F.}(6,10,9)} = \frac{360}{1} = 360.$$

22. $$\text{H.C.F.}\left(\frac{5}{6},\frac{9}{10},\frac{8}{9}\right) = \frac{\text{H.C.F.}\ (5,9,8)}{\text{H.C.F.}\ (6,10,9)} = \frac{1}{90}$$

23. Dividing 256 successively by 2, we get

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

∴ The number of twos in 256! is 1 + 2 + 4 + … + 128

$= 2^8 - 1 = 255$.

24. Let the smallest such number be x.

x must have the forms $17q_1 + 12$ and $24q_2 + 19$ where q_1 and q_2 are whole numbers.

∴ $x = 17\ (q_1 + 1) - 5 = 24\ (q_2 + 1) - 5$

$x + 5 = 17\ (q_1 + 1) = 24\ (q_2 + 1)$

∴ $x + 5$ must be divisible by both 17 and 24 and hence, by their L.C.M. As x is the smallest, $x + 5$ is also the smallest

∴ $x + 5$ = L.C.M. (17, 24)

∴ x = L.C.M. (17, 24) – 5 = (17) (24) – 5 = 408 – 5 = 403.

25. The remainder when any number is divided by 25 is the remainder when the number formed by the last two digits of that number (i.e., 69) is divided by 25 which is 19.

26. The index of each prime factor must be even. If we multiply the number by (5) (7), i.e., 35, the resulting indices are all even.

27. $y! = y\ (y - 1)\ (y - 2)!$

Given

$y! - 20\ (y - 2)! = 0$

$(y - 2)!\ (y.\ (y - 1) - 20) = 0$

$(y - 2)!\ (y^2 - y - 20) = 0$

$(y - 2)! \geq 1$, i.e., it is $\neq 0$

∴ $y^2 - y - 20 = 0$

$(y - 5)\ (y + 4) = 0$

∵ $y > 0$

$y = 5$.

28. The numerator is of the form $a^3 + 3ab^2 + 3a^2b + b^3$, while the denominator is of the form $a^2 + 2ab + b^2$ where a = 2.35 and b = 1.45

$$\frac{a^3 + 3ab^2 + 3a^2b + b^3}{a^2 + 2ab + b^2} = \frac{(a+b)^3}{(a+b)^2} = a + b = 3.8$$

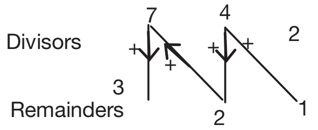
29. Suppose a number x has m digits.

$10^{m-1} \leq x < 10^m$

∴ $10^{3m-3} \leq x^3 < 10^{3m}$

i.e., x^3 has $3m - 2$, $3m - 1$ or $3m$ digits, so if a number has 28, 29 or 30 digits, its cube root has 10 digits.

30.



By solving the equations, we get $x = 10$ and $y = 30$.

∴ The difference between the cost of a book and a pen is 30 – 10 = ₹20.

38. Let us take the two digits as x and y. x is ten's digit and y is the unit's digit, hence, the number itself is equal to $(10x + y)$.

Since sum of the digits is 7,

$x + y = 7$ (1)

When the digits are interchanged, y becomes the $10x + y$ ten's digit and x the units digit. The number then becomes $(10y + x)$, since this number is 27 more than the original number, we get $(10y + x) - (10x + y) = 27 \Rightarrow 9y - 9x = 27$

$\Rightarrow y - x = 3$ (2)

On adding (1) and (2), we get $y = 10/2 = 5$.

By substituting the value of y, we get $x = 2$.

Thus, the number is 25.

39. $12x - 10y - 2 = 0$ (1)

$10x - 10y + 20 = 0 \Rightarrow 10x = 10(y - 2)$

Dividing both sides by 10, $x = y - 2$ (2)

Substituting x as $y - 2$ in equation (1),

$12(y - 2) - 10y - 2 = 0$

$\Rightarrow 12y - 24 - 10y - 2 = 0$

$\Rightarrow 2y = 26 \Rightarrow y = \frac{26}{2} = 13$.

From (2), $x = 11$.

NOTE

Suppose there are two linear equations in two variables, $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where $a_1, b_1, c_1, a_2, b_2, c_2$ are the coefficients of x and y.

(i) The set of equations has a unique solution when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) The set of equations has infinite solutions, when $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(iii) The set of equations has no solutions, when $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

40. Let the cost of each sharpener and eraser be S and E, respectively.

From the data,

$3S + 4E = 25$ (1)

$4S + 3E = 24$ (2)

(1) + (2) ⇒

$7S + 7E = 49$

$\Rightarrow 7(S + E) = 49 \Rightarrow S + E = 7$ (3)

(1) – (3) × (3) ⇒

$$\begin{array}{r} 3S + 4E = 25 \\ \underset{-}{3S} + \underset{-}{3E} = \underset{-}{21} \\ \hline E = 4 \\ \hline \end{array}$$

Substituting $E = 4$, in (3), we get

$3 + 4 = 7 \Rightarrow S = ₹3$

∴ The cost of each sharpener is ₹3 and that of each eraser is ₹4.

41. Let the two-digit number be $10a + b$.

Given $(10a + b) + (10b + a) = k(a + b)$

$\Rightarrow 11(a + b) = k(a + b) \Rightarrow k = 11$ ($\because a + b \neq 0$).

42. Let x be the present age of the man.

20 years ago it was $(x - 20)$ years.

Given $x + 25 = 4(x - 20)$

$\Rightarrow 3x = 105 \Rightarrow x = 35$ years.

43. Let the two digit number be $10x + y$. When its digits are reversed, it becomes $10y + x$.

The difference is $(10x + y) - (10y + x)$.

$= 9x - 9y = 9(x - y)$

Given that the two digits differ by 4, $(x - y) = 4$

∴ The difference $= 9(x - y) = 9 \times 4 = 36$

44. Multiplying the first equation by 3/2 the second equation is obtained.

∴ We have one equation with two unknowns.

∴ (x, y) has infinite values.

45. Let the three-digit number be $100a + 10b + c$.

$a = b + 2$

$c = b - 2$

$a + b + c = 3b = 18 \Rightarrow b = 6$

so $a = 8$ and $c = 4$

Hence, the three-digit number is 864.

46. The difference between a three-digit number and the number formed by reversing its digits = 99 (difference of its first and last digits). As the difference of its first and last digits is 4, the difference of the number and the number formed by reversing its digits = 99 (4) = 396.

47. Let the two digit number be $10a + b$.

$a + b = 12$ (1)

If $a > b$, $a - b = 6$,

If $b > a$, $b - a = 6$

If $a - b = 6$, adding it to equation (1), we get

$2a = 18 \Rightarrow a = 9$

so $b = 12 - a = 3$

$\therefore$ Number would be 93.

if $b - a = 6$, adding it to equation (1),

$2b = 18 \Rightarrow b = 9$

$a = 12 - b = 3$.

$\therefore$ Number would be 39.

$\therefore$ Number would be 39 or 93.

48. Let the cost of each dosa and each idli be ₹ d and ₹ i, respectively.

$2d + 3i = 46$ (1)

$d + 2i = 26$ (2)

(1) – (2) : $d + i = 20$

The cost of 4 idlis and 4 dosas viz $4(d + i) = 4(20)$, i.e., ₹ 80

49. Let the costs of a chair and a table be c and t, respectively. Required cost = $6c + 6t$.

$3c + 4t = 2500$

$4c + 3t = 2400$

By adding the equations and dividing the resulting equation by 7 we get the value of $1c + 1t$.

$6c + 6t = 6(c + t) = 6(700) = 4200$.

50. Let the two-digit number be $10a + b$.

$9a + 8b = 10a + b$

$7b = a$

$0 < a \leq 9$ and $0 < 7b \leq 9$ so $b = 1$

Hence, b is 1, then a = 7, hence, the number is 71.

EXERCISE-2

1. Let cost of each table be x and cost of each chair be y. Then we have the following equations from the given data.

$2x + 3y = 1800$ (1)

$3x + 4y = 2600$ (2)

To solve these two equations, multiply the first equation by 3 and the second by 2 and then subtract one from the other.

We get, $y = 200$

Substituting y in (1), we get

$2x + 600 = 1800$

$\therefore x = 600$

$\therefore$ The cost of each table is ₹ 600 and the cost of each chair is ₹ 200.

2. Let, Venkat's speed be x km/hr and that of Vatsa be y km/hr.

Given, $\dfrac{600}{x} - \dfrac{600}{y} = 2$

$\Rightarrow \dfrac{1}{x} - \dfrac{1}{y} = \dfrac{1}{300}$ (1)

and $\Rightarrow \dfrac{600}{y} - \dfrac{600}{2x} = 4$

$\Rightarrow \dfrac{1}{y} - \dfrac{1}{2x} = \dfrac{2}{300}$ (2)

(1) + (2) $\Rightarrow \dfrac{1}{x} - \dfrac{1}{2x} = \dfrac{1}{300} + \dfrac{2}{300}$

$\Rightarrow x = 50$

From $\dfrac{1}{50} - \dfrac{1}{y} = \dfrac{1}{300} \Rightarrow \dfrac{1}{y} = \dfrac{5}{300} = \dfrac{1}{60}$

$\therefore y = 60$

$\therefore$ Vasta's speed is 60 km/hr.

3. Let the number of marbles with Rahul and Kunal be x and y, respectively.

Given $x + 6 = y - 6$

$x - y = -12$ (1)

$y + 1 = 2\ (x - 1)$ (2)

By solving, we get $x = 15$ and $y = 27$.

Total number of marbles = 15 + 27 = 42.

4. Let the numbers of chocolates with Seoni and Varsha be $7x$ and $9x$, respectively.

Given, $9x - 7x = 14 \Rightarrow x = 7$

Total number of chocolates

$= 7x + 9x = 16x = 16\ (7) = 112$

$\therefore$ Total number of chocolates with them is 112.

5. Let, the cost of each pen, each ruler and each refill be x, y and z, respectively.

$3x + 4y + 5z = 75 \qquad (1)$

$6x + 7y + 10z = 138 \qquad (2)$

$2 \times (1) - (2); \Rightarrow y = 12$

$\therefore\ 3x + y + 5z = (3x + 4y + 5z) - 3y$

$= 75 - 3\ x\ 12 = 75 - 36 = 39$

6. Let the number of apples be a.

If 8 more apples were distributed, number of boys $= \dfrac{a+8}{4}$

Similarly, from the next sentence, number of boys $= \dfrac{a-2}{3}$

$\Rightarrow \dfrac{a+8}{4} = \dfrac{a-2}{3} \Rightarrow a = 32.$

7. Let the numbers of pencils and pens be x and y, respectively.

$\therefore\ 2x + 5y = 50 \qquad (1)$

and $x + y = 16 \qquad (2)$

$(1) - 5 \times (5) \Rightarrow 2x + 5y = 50$

$\underline{5x + 5y = 80}$

$-3x = -30. \quad \therefore\ x = 10$

8. $3x + 4y = 24$

Multiply both the sides with 5. Then,

$15x + 20y = 120 \qquad (1)$

$15x + 20y = 8k \qquad (2)$

To have (1) and (2) as consistent,

$\Rightarrow 120 = 8k, \quad \therefore\ k = 15$

9. Let the present ages of the man and his son be m and s, respectively.

Given, $m - 10 = 2(s - 10) + 35$

$\Rightarrow m - 2s = 25$

or $m = 2s + 25$

i.e., present age of the man is 25 years more than twice his son's present age.

Since we need to find out after how many years will the man's age be twice his son's age, let us assume that it happens after x years.

$\therefore$ After x years $m + x = 2(s + x)$

Substitute $m = 2s + 25$ in above equation

$\Rightarrow 2s + 25 = 2s + x$

$\Rightarrow x = 25$

$\therefore$ After 25 years from now, the age of the man will be twice his son's age.

10. Let the initial amount with Gopi, Murthy and Hari in rupees be x, y and z, respectively.

Gopi gives $\dfrac{x}{2}$ to Murthy

They now have $\dfrac{x}{2}, \dfrac{x}{2} + y$ and z

Murthy gives half of his amount to Hari

They now have

$\dfrac{x}{2}; \dfrac{x}{4} + \dfrac{y}{2}; \dfrac{x}{4} + \dfrac{y}{2} + z$

Hari gives half his amount to Gopi.

Gopi now has $\left(\dfrac{x}{2} + \dfrac{x}{8} + \dfrac{y}{4} + \dfrac{x}{4} + \dfrac{z}{2}\right)$ and this is equal to x.

$\Rightarrow x = \dfrac{x}{2} + \dfrac{x}{8} + \dfrac{y}{4} + \dfrac{z}{2}$

$\Rightarrow \dfrac{3x}{8} = \dfrac{y}{4} + \dfrac{z}{2}$

$\Rightarrow 3x = 2y + 4z = 2 \times 45 = 90$, as $(y + 2z)$ is given equal to 45.

$\Rightarrow x = 30$

11. Let Prakash and his son's present ages be x years and y years, respectively.

Four years from now, Prakash will be $(x + 4)$ years and son will be $(y + 4)$ years.

$\therefore\ x + 4 = 4(y + 4)$

$\Rightarrow x - 4y = 12 \qquad (1)$

Twelve years from now, Prakash will be $x + 12$ and his son $y + 12$.

$\therefore\ x + 12 = \dfrac{5}{12}(y + 12)$

$\Rightarrow 2x - 5y = 36 \qquad (2)$

$[(1) \times 2] - (2)$ gives $-3y = -12$, $y = 4$

By substituting $y = 4$ in (1), we get $x = 28$.

12. After purchasing 4 apples and 5 mangoes, the man will be left with 1/4 of what he initially had, which is ₹20. He had ₹80 to start with.

With ₹80, the man can purchase 16 apples. Each apple costs $\frac{80}{16} =$ ₹5

With ₹80, if the man can purchase 10 mangoes, each mango costs = $\frac{80}{10} =$ ₹8.

∴ The difference in the prices of an apple and a mango is ₹3.

13. Let the number of chocolates received by A, B, C and D be a, b, c, and d, respectively.

$a + b + c + d = 225$ (1)

$a + d = 2\ (b + \mathrm{c})$

Substituting $a + d$ as $2(b + c)$ in equation (1), we get

$3(b + c) = 225 \Rightarrow b + c = 75$

As $b = c + 15$,

$c + 15 + c = 75$

$c = 30$.

14. Let the three-digit number be xyz.

$x + y + z = 9$ (1)

$xyz - 99 = zyx$

$\Rightarrow 99x - 99z = 99$

$\Rightarrow x - z = 1$ (2)

$(1) + (2) \Rightarrow 2x + y = 10$.

∴ (x, y) can be (1, 8), (2, 6), (3, 4), (4, 2) or (5, 0).

∴ xyz can be 180, 261, 342, 423 or 504. The least number satisfying the second condition is 261.

15. Let Ganesh's present age be g years.

Govind's present age = $4g$ years.

$4g + 20 = 2\ (g + 20)$

$g = 10$

16. Let the total number of employees be x.

Given two thirds of them are software professionals and of these four-fifths are males

i.e., $\frac{4}{5}\left(\frac{2}{3}x\right) = 240 \Rightarrow x = 450.$

17. Let the present ages of Ram, Sita and their daughter be r, s and d, respectively.

$\frac{r+s+d}{3} = 35$

$\Rightarrow r + s + d = 105$ (1)

$s + 15 = r + \mathrm{d}$

Substituting $r + d$ as $s + 15$ in (1), we get $2s + 15 = 105$

$\Rightarrow 2s = 90 \Rightarrow s = 45$ years

18. Let the cost of a burger be ₹ p, and that of a pizza be ₹ q.

$7p + 8q = 780$ (1)

$12p + 5q = 945$ (2)

$12 \times (1) - 7 \times (2) \Rightarrow 84p + 96q = 9360$

$\underline{84p + 35q = 6615}$

$61q = 2745$

∴ $q = 45$

∴ The cost of each pizzas is ₹45.

19. Let the number of questions he answered correctly be x. Then for $50 - x$ questions were answered wrongly.

Total marks scored by the student

$= x \times 3 - (50 - x)1 = 90 \Rightarrow x = 35.$

20. The equations have infinite solutions.

$\frac{4}{k} = \frac{k-10}{24} = \frac{2}{8}$

$\frac{4}{k} = \frac{2}{8} \Rightarrow k = 16$

21. Let the present ages of the father and son be x years and y years, respectively.

We now have,

$x = 10y$ (1)

$x + 6 = 4(y + 6) = x - 4y = 18$

Solving (1) and (2), we get $x = 30$ and $y = 3$.

Let father's age be twice the son's age in 'p' years.

Then we have $30 + p = 2(p + 3) \Rightarrow p = 24$

So, in 24 years, the father will be twice as old as his son.

22. Let the number of boys be B and the number of girls be G. Since there is a difference in the number of brothers for the narrator and his sister, the narrator has to be a boy.

For a boy, no of brothers = $b - 1$ and number of sisters $= g$; so $b - 1 = 3\mathrm{g}$.

For a girl, no of brothers = b and the number of sisters = $g - 1$; so $b = 4(g - 1)$. Solving the 2 equations, we get the $(b, g) = (16, 5)$. $\therefore\ b + g = 21$.

23. Let the number of 25 paise and 20 paise coins be a and b, respectively.

$25a + 20b = 1400$

$25b + 20a = 1300$

Adding both equations, $45\ (a + b) = 2700$

$\Rightarrow a + b = 60$

24. If we multiply $4x - 3y + 6 = 0$ with -2, we get $6y - 8x - 12 = 0$ which differs from the second equation, w.r.t., constant. $\therefore$ for $k = 6$ the system is inconsistent.

25. Let the counter price of each ticket be x.

Cost of 2 counter tickets = $2 \times x$.

Cost of 2 extra tickets = $2(x + 50)$

Total amount = $4x + 100$

Total money they spent = $4 \times 60 = 240$.

$\therefore\ 4x + 100 = 240$.

$\Rightarrow x = 35$.

26. Let the number of 25 paise and 50 paise coins with Kunal be x and y, respectively.

Given, $a - b = 20$ (1)

and $50b - 25a = 400$ (2)

Solving (1) and (2),

$a = 56$ and $b = 36$

$\therefore$ The amount with Kunal is $25(56) + 50(36) = 3200$ paise, i.e., ₹ 32.

27. Let the length of the middle sized piece be x m

Given: length of the longest piece = 3(length of middle sized piece) = $3x$

Also, the length of the shortest piece = (length of the longest piece) – 34 m = $3x - 34$

As the length of the rope is 64 m,

$3x + x + 3x - 34 = 64 \Rightarrow 7x = 98 \Rightarrow x = 14$

28. $2x + 3y + 2z = 23$ (1)

$3x - 2y + 3z = 28$ (2)

From (1) and (2) $13y = 13 \Rightarrow y = 1 \Rightarrow x + z = 10$.

29. Let Praveen's present age be x years and that of Mahesh be y years.

$x = 2(y - 4) \Rightarrow x - 2y + 8 = 0$ (1)

$x + 8 = 2y \Rightarrow x - 2y + 8 = 0$ (2)

Since the two equations are the same, the sum of their present ages cannot be uniquely determined.

30. P, Q and R are successive even positive integers in the ascending order.

$\Rightarrow R - Q = 2$ and $Q - P = 2$

$\Rightarrow R - P = 4$

$4R = 5P + 4$

$\Rightarrow 4R - 5P = 4$

$\Rightarrow 4R - 4P - P = 4$

But $R - P = 4$

$\therefore 4(R - P) - P = 4$

$\Rightarrow 4(4) - P = 4$

$\Rightarrow P = 12$

$\therefore\ Q = P + 2 = 12 + 2 = 14$

31. $x + 2y + 3z = 14$ (1)

$2x + y + 2z = 10$ (2)

$3x + 3y + 4z = 21$ (3)

Multiply equation (1) with 2 and subtract equation (2) from the resulting equation to eliminate 'x' and obtain an equation in 'y' and 'z'.

$$\begin{array}{l} 2x + 4y + 6z = 28 \\ \underline{\underset{-}{2x} + \underset{-}{y} + \underset{-}{2z} = \underset{-}{10}} \\ 3y + 4z = 18 \end{array} \qquad (4)$$

Now from any two other equations eliminate x.

For example take equations (1) and (3)

Multiply equation (1) with 3 and subtract equation (3) from the resulting equation

$$\begin{array}{l} 3x + 6y + 9z = 42 \\ \underline{\underset{-}{3x} + \underset{-}{3y} + \underset{-}{4z} = \underset{-}{21}} \\ 3y + 5z \quad\ = 21 \end{array} \qquad (5)$$

Now solve equations (4) and (5) for 'y' and 'z'

$$\begin{array}{l} 3y + 4z = 18 \\ \underline{\underset{-}{3y} + \underset{-}{5z} = \underset{-}{21}} \\ -z \quad = -3 \end{array}$$

Substituting z in (4), we get $y = 2$

Substitute $y = 2$ and $z = 3$ in any of (1), (2) or (3) to get the value of x.

$x + 2(2) + 3(3) = 14$

$\Rightarrow x + 4 + 9 = 14$

$\Rightarrow x + 13 = 14 \Rightarrow x = 1$

32. Let the price per kg of Oranges, Mangoes, Bananas and Grapes be ₹ *R*, ₹ *M*, ₹ *B* and ₹ *G*, respectively.

Given that

$5R + 2M = 310$ (1)

$3M + 3.5B = 230$ (2)

$1.5B + 5G = 160$ (3)

(1) + (2) + (3) $\Rightarrow 5R + 5M + 5B + 5G = 700$

$\therefore\ 10R + 10M + 10B + 10G = 2 \times 700 = 1400$

33. Let the number be $10x + y$

$x + y = 3(x - y)$ ($\because$ $xy > yx$) $\Rightarrow 2x = 4y$

or, $x = 2y$ (1)

Also, $(10x + y) - (10y + x) = 36$

$\Rightarrow 9x - 9y = 36 \Rightarrow x - y = 4$ (2)

$\therefore\ y = 4$ and $x = 8$

$\therefore$ The number is 84.

34. Let the costs of each pencil, each ruler and each eraser be *p*, *r* and *e*, respectively.

$3p + 5r + 7e = 49$ (1)

$5p + 8r = 11e = 78$ (2)

2 x (2) – 3 x (1) $\Rightarrow$

$10p + 16r + 22e = 156$

$9p + 15r + 21e = 147$

$p + r + e = 9$

$\therefore$The cost of 1 pencil, 1 ruler and 1 eraser is ₹ 9

35. Let the present ages of Ajay and Bala be a years and *b* years, respectively.

$a - 20 + b - 20 = \frac{5}{9}(a + b)$

$a + b = 90$ (1)

$a - b = 20$ (2)

Solving (1) and (2), $a = 55$

36. Here, $a_1/a_2 = b_1/b_2$

Thus for any other value of *k* except 15, we will have a case of

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

So, the only value of *k* that makes the equations consistent is 15.

37. Let, the present age of the man be *x* years and that of the son be *y* years.

$x - 4 = 3(y - 4) \Rightarrow x - 3y + 8 = 0$ (1)

and $x + 8 = 2(y + 8) \Rightarrow x - 2y - 8 = 0$ (2)

Solving (1) and (2), we get

$x = 40$ and $y = 16$

38. Let, *x* and *y* be the amounts with Amit and Sunil, respectively. Given,

$x - 40 = y + 40$

$\Rightarrow x - y = 80$ (1)

$x + 10 = y - 10 + 100$

$\Rightarrow x - y = 80$ (2)

Since the two equations are the same, the value of *x* or *y* cannot be uniquely determined.

39. Let the present ages of *P* and *Q* be *x* and *y* years, respectively.

Then, $x + y = 84$ (1)

Also, $x - 6 = 2(y - 6)$

$x - 2y = -6$ (2)

By solving the equations (1) and (2), we get $x = 54$ and $y = 30$

$\therefore$ Difference = $x - y = 24$.

40. Let the combined present age of the parents of the girl be *x* and the present age of the girl be *g*.

Given, $x = 5g$

Also, $(x + 16) = 4(g + 8)$

$5g + 16 = 4g + 32$

$g = 16$

$\therefore\ x = 5\ (16) = 80$

The combined age of the parents when the girl was born $= 80 - (16 + 16) = 80 - 32 = 48$ years.

41. Given that, $(a + b) = 2(a - b)$ (1)

$a = 3b$, *a* must be a multiple of 3 (as *b* is an integer). (a, b) can be (3, 1), (6, 2) or (9, 3). So, there are three numbers which satisfy the given condition.

42. When the woman was 29, the son was born and at that time the daughter was 3. Therefore, the woman was (and is) 26 years older than her daughter. As the woman is 3 times as old as the daughter, she is 39 and the daughter is 13. Therefore, the son is 10.

43. Let the fraction be $\frac{x}{y}$

When both the numerator and the denominator are increased by 2 each, we have

$$\frac{x+2}{y+2} = \frac{3}{5} \Rightarrow 5(x+2) = 3(y+2)$$

$\Rightarrow 5x - 3y = -4$ (1)

When both the numerator and the denominator are increased by 1 each, we have

$$\frac{x+1}{y+1} = \frac{1}{2} \Rightarrow 2(x+1) = y+1$$

$\Rightarrow 2x - y = -1$ (2)

$[(1) - (2) \times (3)]$ gives $-x = -4 + 3 \Rightarrow x = 1$. Substituting this value in (2), $y = 3$ is obtained.

Hence, the fraction is 1/3.

44. Let the costs of a pen, a pencil and an eraser be *a*, *b* and *c*, respectively.

Given: $a + b + 3c =$ ₹140 (1)

$5a + 3b + c =$ ₹320 (2)

We need the value of $3a + 2b + 2c$.

$\frac{1}{2}[(2) + (1)]$ gives

$\Rightarrow 3a + 2b + 2c =$ ₹230

45. The duration from 2:00 a.m to 8: 00 a.m , which is 360 minutes, is the sum of 3 parts – t minutes, 40 minutes and $3t$ minutes.

$\therefore\ t + 40 + 3t = 360$ min

$4t + 40 = 360$ min

$4t = 320$ min $\Rightarrow t = 80$ min

Present time = 8:00 a.m – 80 min = 6:40 a.m

46. Let the cost of 1 kg of tomatoes be ₹ x and the cost of 1 kg of potatoes be ₹ y

$6x + 7y = 190$ (1)

$8x + y = 170$ (2)

$(1) \times 4 - (2) \times 3$

$24x + 28y = 760$ (3)

$24x + 3y = 510$ (4)

$(3) - (4)\ 25y = 250,\ y = 10.$

Substitute $y = 10$ in $-(-2)$, $x = 20$.

$\therefore$ The cost of 1 kg of tomatoes is ₹20.

47. Let the two digit number be xy.

Using the first statement,

$x - y = 4$ (1)

or $y - x = 4$ (2)

Using the second statement, $10x + y + 10y + x = 110$.

$\therefore\ 11x + 11y = 110$

$x + y = 10$ (3)

From (1) and (3), $xy = 73$.

From (2) and (3), $xy = 37$.

$xy = 73$ or 37.

Solutions for questions 48 and 49:

Let the costs (in ₹) of a cup of ice-cream, a burger and a soft drink be *C*, *B* and *S*, respectively.

48. $3C + 2B + 4S = 128$ – I

$2C + 1B + 2S = 74$ – II

Multiply II by 2 and then subtract I from it

i.e., $4C + 2B + 4S = 148$

$-\ (3C + 2B + 4S = 128)$

$\therefore\ C = 20$

$\therefore$ The cost of 1 cup of ice-cream = ₹20

49. Substituting the value of *C* in equation II of the above problem, we have,

$B + 2S = 34$ – III

5 times equation III is $5B + 10S = 170$

$\therefore$ The cost of 5 burgers and 10 soft drinks = ₹170

50. Let the number to be multiplied be x.

$$\frac{4}{7}x - \frac{4}{17}x = 840 \Rightarrow 4x\left[\frac{1}{7} - \frac{1}{17}\right] = 840$$

$$x\left[\frac{17-7}{119}\right] = 210 \Rightarrow 10x = 210\ x\ 119$$

$\therefore\ x = 2499$

EXERCISE-3

1. The number of chocolates with Balu at different stages can be tabulated as below.

	Numbers given	Balu has
		78
To eldest son half + 3	39 + 3	36
To second eldest son (one third + 4)	12 + 4	20
To youngest son (one fourth + 4)	5 + 4	11

After giving 4 more than one-fourth to the youngest son he is left with 11 which means that Balu was left with 4 less than three – fourths. (11 + 4) is three fourths and hence, 5 is one fourth. He has 20 chocolates before giving to his youngest son. Similarly he has 36 and 78 before giving to his second eldest son and his eldest son, respectively.

2. Let the number of chocolate boxes with Ramu in the beginning be x.

He sold $\left(\frac{1}{2}x+\frac{1}{2}\right)$ boxes to the first customer.

He would be left with $\left(\frac{x}{2}-\frac{1}{2}\right)$ boxes. He sold $\frac{1}{2}\left(\frac{1}{2}x-\frac{1}{2}\right)+\frac{1}{2}=\left(\frac{1}{4}x+\frac{1}{4}\right)$ boxes to the second customer. He would be left with $\left(\frac{1}{4}x-\frac{3}{4}\right)$ boxes. He sold $\frac{1}{2}\left(\frac{1}{4}x-\frac{3}{4}\right)+\frac{1}{2}=\left(\frac{1}{8}x+\frac{1}{8}\right)$ boxes to the third customer. He would be left with $\left(\frac{1}{8}x-\frac{7}{8}\right)$ boxes.

It can be seen that to the nth customer he would have sold $\left(\frac{1}{2}\right)^n(x+1)$ boxes. After that he would be left with $\left(\frac{1}{2}\right)^n x-\left(1-\left(\frac{1}{2}\right)^n\right)$ boxes.

Since $n = 10$, $\frac{x}{1024}=1-\frac{1}{1024}$

$\Rightarrow x = 1023$

3. Let x and y be the units and ten's digits of the two digit number.

Given $y > x$ and $10x + y + 10y + x = 2[9\,(y-x)] + 2$

$\Rightarrow 11x + 11y = 18y - 18x + 2$

$29x = 7y + 2$

$x = 2$ and $y = 8$ is the only solution set that satisfies this equation

(both lie between 0 and 9)

$\therefore$ the number is 82

4. Let the prices of the four varieties of pens be p_1, p_2, p_3 and p_4 and that of the four varieties of pencils be c_1, c_2, c_3 and c_4.

If c_1 is the price of the cheapest pencil and c_2, c_3 and c_4 are the prices of three of the varieties of the pens, we have

$p_2 + p_3 + p_4 = 2(c_2 + c_3 + c_4)$

As $c_1 = 1$, $c_2 + c_3 + c_4 =$ ₹11

$\therefore p_2 + p_3 + p_4 = 22$

As $p_1 + p_2 + p_3 + p_4 = 45$, we have $p_1 =$ ₹23, which has to be the highest price (as the sum of the other 3 prices is 22).

***Solutions for questions 5 and 6*:**

Let, the number of pens, erasers and rulers be p, e and r, respectively.

$p > e > r$ and $p \geq 10$, $e \geq 10$, $r \geq 10$

$p + e + r = 35$

If $r = 10$, there are two possibilities, $p = 14$, $e = 11$ or $p = 13$, e = 12.

r cannot take a value greater than or equal to 11, since if $r = 11$, $p + e = 24$, which is not possible.

$\therefore\ r = 10$

5. $r = 10$.

6. Minimum amount spent

$= 13 \times 20 + 12 \times 5 + 10 \times 2 = 340$

7. All the amounts are in rupees. Let us assume that he started the game with rupees A. Amount with him at the end of the first round = ₹ $(2A - x)$.

Amount with him at the end of the second round = ₹ $2(2A - x) - x =$ ₹ $(4A - 3x)$

Amount with him at the end of the third round = ₹ 2(4A − 3x) − x = ₹ (8A − 7x).

It can be seen that the amount with him at the end of the nth round = ₹ $(2nA - (2n - 1)x)$

The amount with him at the end of the 10th round = $1024A - 1023\ x = 1023 \Rightarrow \frac{A}{x+1} = \frac{1023}{1024}$

∴ $x + 1$ must be divisible by 1024

∴ least possible value of $x = 1023$

Sum of its digits is 6.

Solutions for questions 8 and 9:

Let the actual number of toys sold be 'ab'.

As the stock left showed 81 items more than what it actual was, the mistaken number of items sold must be 81 less than the actual number sold.

∴ 'ab' − 'ba' = 81 ⇒ $(10a + b) - (10b + a) = 81$

$\Rightarrow a - b = 9$

∴ $a = 9$ and $b = 0$

8. There is only *one* possibility.

9. Actual selling price = reverse of $\frac{486}{\text{'}ba\text{'}}$

$= \text{Reverse of } \frac{486}{9} = ₹\,45$

10. Let the three digit number be $100x + 10y + z$

Now, $(100x + 10y + z) - (100z + 10y + x) = 297$

$\Rightarrow 99(x - z) = 297$

$\Rightarrow x - z = 3$ (1)

Also, $y + z = x - z = 3$

$x = 2z$ (2)

Substituting (2) in (1) gives $z = 3 \Rightarrow x = 6$ and $y = 0$

∴ The required number is 603.

11. Let '$abcdef$' be any 6-digit number and let N be the reverse.

$M = 10^5a + 10^4b + 10^3c + 10^2d + 10^1e + f$

$N = 10^5f + 10^4e + 10^3d + 10^2c + 10^1b + a$

∴ $M + N = (10^5 + 1)a + (10^4 + 10)b + (10^3 + 10^2)c + (10^2 + 10^3)d + (10 + 10^4)e + (1 + 10^5)f$

$= 1000{,}001\ (a + f) + 10{,}010(b + e) + 1100\ (c + d)$

∴ $M + N$ has to be a multiple of 11.

The 11's remainder is the same as that of (6 + 7 +0) − (0 + 6 +9) = − 2 or 9.

All the other numbers are multiples of 11. It is possible for $M + N$ to be any of the other numbers. But it cannot be 906706.

12. $14 + \frac{x}{4} + \frac{x}{4} = 4x$

$\Rightarrow 4x - \frac{x}{2} = 14$

$\Rightarrow \frac{7x}{2} = 14$

$\Rightarrow x = 4$

∴Present age $= 14 + \frac{x}{4} = 15$

5x years from now, his age will be 15 + 5 × 4 = 35 years.

Solutions for questions 13 to 14:

Let the number of toys actually sold = $100x + 10y + z$

Now $(100x + 10y + z) - (100z + 10y + x) = 792$

$\Rightarrow 99(x - z) = 792 \Rightarrow x - z = 8$

∴ $x = 9$, $y = 1$ is the only possible solution set, since y is a non-zero digit.

And also as $y = z$, $z = 1$.

∴ The required number is 911.

13. From the above, the number of toys actually sold is 911.

14. Faulty selling price $= \frac{5117}{119} = 43$.

∴Actual selling price of each toy = ₹ 34

15. Actual total sales = 911 × 34 = ₹ 30, 974

16. The equation dependent on the first two given equations can be written as $(3x + 2y - 7z - 56) + k(5x + 3y + z - 16) = 0$. This can be written as

$x(3 + 5k) + y(2 + 3k) + z(-7 + k) + (-56 - 16k) = 0$ (1)

As the given equations are dependent, the corresponding coefficients of the third equation and equation (1) are proportional.

$$\frac{p}{3+5k} = \frac{12}{2+3k} = \frac{-19}{-7+k} = \frac{-200}{-(56+16k)}$$

$$\Rightarrow \frac{12}{2+3k} = \frac{-19}{-7+k} \Rightarrow k = 2/3.$$

Put the value of k in (1), we get,

$$x\left(3+\frac{10}{3}\right) + y\left(2+\frac{6}{3}\right) + z\left(-7+\frac{2}{3}\right) - \left(56+\frac{32}{3}\right) = 0.$$

$19x + 12y - 19z = 200$. So, $p = 19$.

17. Let the four digit number be $abcd$

$a + d = b + c$ (1)

$b + d = 4(a + c)$ (2)

$a + b + c + d > 10$, i.e., $b + d + a + c > 10$

$4(a + c) + a + c > 10 \Rightarrow a + c > 2$.

Least $a + c$ is 3. When $a + c$ is 3, $b + d$ is 12.

$\therefore$ a + b + c + d is 15. But then (1) would be violated $\therefore$ $a + c \neq 3$.

If $a + c$ is 4, $b + d$ is 16. $\therefore$ a + b + c + d is 20.

and $a + d = b + c = 10$

$(a, b, c, d) = (a, 6 + a, 4 - a, 10 - a)$

If $a + c$ is 4, $b = 6 + a$.

$\therefore$ b is 7, 8 or 9

(but not 6 as $a \neq 0$)

If $a + c$ is 5, $b + d = 20$. But this is not possible ($\because$ $b \leq 9$ and $d \leq 9$. $\therefore$ max $(b + d) = 18$)

$a + c$ cannot be 5 or more.

$a + c$ must be 4.

$\therefore$ b can be 7, 8 or 9 but not 6.

18. Let the number of silver pendants with Laxmilal be x.

Then the number of silver pendants with Kuberjain will be $36 - x$. Also, let Kuberjain give y pendants to Laxmilal.

Then $x + y = 5\ (36 - x - y)$

$\Rightarrow x + y = 30$ (1)

If Laxmilal gives y pendants to Kuberjain, then

$x - y = 36 - x + y$

$\Rightarrow x - y = 18$ (2)

From (1) and (2), $x = 24$.

19. Let the present age of 'X' be x years.

$\therefore$ present age of Y = $(63 - x)$ years.

Present age of Y = past age of 'X' = $63 - x$

The difference between their past and present ages

$= x - (63 - x)$

$= 2x - 63$

$\therefore$ Past age of Y = Present age of Y – difference of ages.

$= (63 - x) - (2x - 63) = 126 - 3x$

Present age of X = 2(past age of Y)

$x = 2(126 - 3x) = 252 - 6x$

$\Rightarrow 7x = 252 \Rightarrow x = 36$.

20. Given equations are

$x + 3y - 4z = a$

$4x + y - 5z = b$

$x + y - 2z = c$

If a, b and c can be expressed one in terms of the other, then they will have at least one solution. By observation of choices,

$11c = 11x + 11y - 22z$

$3a = 3x + 9y - 12z$

$2b = 8x + 2y - 10z$

$\therefore 11c = 3a + 2b$

i.e., $3a + 2b - 11c = 0$.

21. Let the ages of Shreya and Lata be s and l, respectively. Shreya was as old as Lata is exactly $(s - l)$ years ago. Then Lata would have been $l - (s - l) = 2l - s$ years old

Given, $s = 3(2l - s) \Rightarrow s = \frac{3}{2}l$ (1)

and $s + l = 80$ (2)

Solving (1) and (2), $s = 48$, $l = 32$

$\Rightarrow s - l = 16$

22. Let the number be ab

$ab = 4(a + b) - 12$

$10a + b = 4(a + b) - 12$

$2(a + 2) = b$

If $a = 1$, $b = 6$

If $a = 2$, $b = 8$

If $a \geq 3$, the digit b exceeds 9.

$\therefore$ab could be 16 or 28

Only 16 satisfies the second condition given.

23. Let us say the person paid a 1 Besos, b 5 Besos and c 20 Besos to settle the bill. Then, the bill amount (in Besos) $= a + 5b + 20c = 49$

Since he has at least one of each type, so a, b $c \geq 1$

If $c = 1$, a + $5b = 29$.

If $c = 2$,

$a + 5b = 9$.

If $c \geq 3$, $a + 5b$

would be negative which is not possible.

$\therefore$ c = 1 or 2. We consider each of these possibilities below.

Possibility 1: c = 1

$a + 5b = 29$

$\therefore$ $(a, b) = (24, 1), (19, 2), (14, 3), (9, 4), (4, 5)$

$\therefore$ (a, b) has 5 possible values.

Possibility 2: $c = 2$

$a + 5b = 9$

$\therefore$ $(a, b) = (4, 1)$

$\therefore$ (a, b) has 1 possibility.

(a, b) has a total of 6 possibilities.

24. Let the gambler start with an amount x and after the first round he had $(3x – p)$.

After second round he had $[2(3x – p) – 3p]$. After third round he had $4\ [2(3x – p) – 3p] – 2p$ viz 0 (given). (1)

$p + 2p + 3p = 360$ (given) (2)

from (2) $\Rightarrow p =$ ₹60 (3)

From (3) and (1) $x =$ ₹55

25. For the equations to have infinitely many solutions.

$\frac{2}{k} = \frac{k-2}{12} = \frac{1}{3}$ must be satisfied

$\frac{2}{k} = \frac{1}{3} \Rightarrow k = 6$

26.

	A	B
Present	28	x
Past	3/5 x	20

$28 – 3/5\ x = x – 20$

Thus, $x + 3/5\ x = 48$

$8/5x = 48$

$x = 30$

27. We have to decide whether y is greater than 40 or less than 40.

$2y < 50.\ \therefore y < 25.$

$y – 40 + 100 = 2x$

$y + 60 = 2x$ (1)

$x – 8 – 1 = 2y$

$x – 9 = 2y$ (2)

Solving (1) and (2), $x = 37$ and $7y = 14$.

The amount is ₹37.14.

28. Let the number of roses blooming in the month of January be x. The number for successive months

Month	Number	Month	Number
Jan	x	Apr	$\frac{x}{4} + 30$
Feb	$\frac{x}{2}$	May	$\frac{x}{4} + 90$
Mar	$\frac{x}{2} + 60$	June	$\frac{x}{8} + 45$

$\therefore$ Number of roses blooming in June is

$\frac{x}{8} + 45 = 120 \Rightarrow x = 600$

29. As number of roses that bloomed in June is 120, in successive

June	July	August	September
120	180	90	150

$\therefore$ 30 more flowers bloomed in September.

30. Let the amounts with Prakash, Sameer, Ramesh and Tarun be ₹ p, ₹ s, ₹ r and ₹ t, respectively.

$p + s + r + t = 240$

$p = \frac{1}{2}(s + r + t) = \frac{240 - p}{2}$

$p = \frac{1}{3}\ (240) = 80$

So, half of the total amount with the others has become one-third of the total amount.

Similarly, $s = \frac{1}{4}(240) = 60$ and $r = \frac{1}{5}\ (240) = 48$

$t = 240 – (p + s + r) = 52$

31. Let the number of correct answers wrong answers, unattempted questions be C, W, U, respectively.

Then, $C + W + U = 120$ (1)

$4C – 2W – U = 228$ (2)

$4 \times (1) – (2) => 6W + 5U = 252$. As W should be maximum, U should be minimum.

For $U = 0$, $W = 42$.

Hence, the number of questions for which he gave a wrong answer can at the most be 42.

32. Let $\frac{1}{x+2y} = p$ and $\frac{1}{3x+4y} = q$

Then, $15p – 11q = 2$ (1)

$\frac{5}{2}\ p + 22q = \frac{5}{2}$

$\Rightarrow 5p + 44q = 5$ (2)

(1) – (2) × 3

$\Rightarrow \quad 15p - 11q = 2$

$15p + 132q = 15$

$-143q = -13$

$\Rightarrow q = 1/11$

From (1), $15p - 11 \times \frac{1}{11} = 2$

$15p = 2 + 1 \Rightarrow p = 1/5$

But $p = \frac{1}{x+2y}, q = \frac{1}{3x+4y}$

$\Rightarrow \frac{1}{x+2y} = 1/5 \Rightarrow x + 2y = 5$ (3)

$= \frac{1}{3y+4y} 1/11 \Rightarrow 3x + 4y = 11$ (4)

(4) – 2 × (3)

$\Rightarrow \quad 3x + 4y = 11$

$2x + 4y = 10$

$x = 1$

$3\ (1) + 4y = 11 \Rightarrow y = 2$

$\therefore x = 1, y = 2$

33. Let s be the present age of Mr. Smith and a, b, c be the present ages of Andy, Bandy and Candy, respectively. Let y be the present age of Mrs. Smith.

$2\ [(a + b + c) - 21] = s - 7$ (1)

$2\ (a + b + c) = s + y$ (2)

Subtracting (1) from (2)

$42 = y + 7 \Rightarrow y = 35$ years

34. The left pan weighs 0.6 kg and the right one weighs 0.95 kg. When the pans level, the total weights on the two sides are equal.

The two weighings are shown in the tables below. Let the actual weight of the rice be r kg.

L	R	L	R
0.6	0.95	0.6	0.95
r	ab	$ba + 18.7$	r

'ab' is 0.35 less than r.

'ba' +18.7 is 0.35 more than r.

$\therefore$ 'ba' +18.7 is 0.7 more than ab

i.e., 'ba' + 18 is equal to 'ab' (i.e., $a > b$)

$\therefore 10b + a + 18 = 10a + b$

$\Rightarrow a - b = 2.$

($\therefore$ ab could be 20 ,31,42,53,64,75,86, or 97)

The weight of the rice is ab + 0.35, i.e., it could be 20.35, 31.35,... or 97.35.

From the options given, it can be 53.35.

35. Let the three digit number be abc. Given, $abc - cba = 396$.

Hence, $99(a - c) = 396 \Rightarrow \quad a - c = 4$

Also, $a - b = b - c$ ($a - b \neq c - b$ because $a \neq c$) $\Rightarrow \quad a + c = 2b.$

If b is less than or equal to 2, we get c as zero or negative. For b = 3, 4, 5, 6 or 7, we've the numbers 531, 642, 753, 864 and 975, respectively. Hence, there are five such possible numbers.

36. $4x + 5y = 32$ (1)

and $6x + 7.5y = \text{k}$ (2)

As (1) and (2) are not inconsistent, i.e., consistent, equation (1) × 1.5 must be equal to equation (2).

$\therefore$ Value of $k = \frac{32 \times 3}{2} = 48$

37. Let the number of questions answered correctly, answered wrongly and unanswered by the student be C, W and U, respectively.

$C + W + U = 120$ and $C - \frac{1}{3}W - \frac{1}{6}U = 60.$

$C + W + U - (C - \frac{1}{3}W - \frac{1}{6}U) = 120 - 60$, i.e., $\frac{4}{3}W + \frac{7}{6}U = 60$

Multiplying both sides by 6, we get $8W + 7U = 360$.

As 360 is divisible by 8, $8W + 7U$ must also be divisible by 8.

$\therefore$ $7U$ must be divisible by 8. If U is not divisible by 8, $7U$ will not be divisible by 8. But $7U$ is divisible by 8.

$\therefore$ U must be divisible by 8 (1)

$7U$ cannot exceed 360.

$\therefore$ U cannot exceed $\frac{360}{7}$, i.e., $51\frac{3}{7}$ (2)

From (1) and (2), maximum value of U is 48. Also, when U is maximum, W is minimum.

$\therefore$ Min $(W) = \frac{360 - 7(48)}{8} = 3.$

38. Let the four-digit number be *abcd*.

$b + c = a + d$ (1)

$b + d = 5(a + c)$ (2)

$\Rightarrow a + b + c + d = 6(a + c)$

Given that $a + b + c + d = 6(a + c) = 18$

$\Rightarrow a + c = 3 \Rightarrow b + d = 15$

$\Rightarrow (b, d) = (6, 9), (7, 8), (8, 7), (9, 6)$ and (a, c)

$= (1, 2), (2, 1), (3, 0)$ as $a \neq 0$.

But according to (1), we have $a + d = b + c$

$\Rightarrow (a, b, c, d) = (1, 7, 2, 8), (2, 8, 1, 7), (3, 9, 0, 6)$

∴The hundreds digit can be 7, 8 or 9.

So, their sum is 7 + 8 + 9 = 24

39. Let the number of males in Chotasansthan be *m*. Then, number of males in Badasansthan is $x + 5114$.

Also, the number of females in Chotasansthan is $2x$.

Number of females in Badasansthan is $2x – 9118$. Also, when compared to males, as females are 3004 less in Badasansthan, it has $(x + 5114) – 3004$ females, i.e., $x + 2110$.

$\therefore 2x – 9118 = x + 2110 \ \therefore x = 11{,}228$.

∴Number of females in Badasansthan

= 11,228 + 2110 = 13,338.

40. Adding the given equations, we have $8(p + q + r + s + t + u) = 120$

$\therefore\ p + q + r + s + t + u = 15$

$3p + q + r + s + t + u - (p + q + r + s + t + u) = 0$

$\therefore\ P = 0$

$\therefore\ (p)(q)(r)(s)(t)(u) = 0$

41. Let the number of ₹2, ₹5 and ₹10 notes in the bag be denoted by *a, b* and *c,* respectively.

$a + b + c = 120$ (1)

$2a + 5b + 10c = 760$ (2)

$2a + 5(2b) + 10c = 960$ (3)

$\Rightarrow 2a + 10b + 10c = 960$

$\Rightarrow 2a + 10(120 – a) = 960$

$a = 30$

Subtracting (2) from (3),

$5b = 200$

$b = 40$

Substituting a and b in (1), $c = 50$.

42. Let $100a + 10b + c$ be the number

$\therefore\ a – b = b – c$ (or) $a – b = c – b$

$\Rightarrow a + c = 2b$ (or) $a = c$

When $a + c = 2b$ and $a + b + c = 9$, $b = 3$ and $a + c = 6$.

Hence, a can have values 1 to 6, i.e., six possible numbers.

When $a = c$, the possible numbers are 171, 252, 333 and 414, i.e., 4 possible numbers. But 333 is common to both.

The number of possible numbers = 6 + 4 –1 = 9

43. (i) If the equations given have infinite solutions,

$$\frac{p}{2} = \frac{q}{3} = \frac{66}{8}$$

$$\therefore\ p = \frac{66}{4} \text{ and } q = \frac{99}{4}$$

$\therefore\ 4(p + q) = 165.$

(ii) The equation given will have no solution if

$$\frac{p}{2} = \frac{q}{3} \text{ and neither } \frac{p}{2} \text{ nor } \frac{q}{3} \text{ is } \frac{66}{8}$$

As $q = 9$, $p = 6$

NOTE

The equations given will have a unique solution if $\frac{p}{2} \neq \frac{q}{3}$.

44. Eliminate ‘*x*’ from equations (1) and (2)

The three given equations are as follows:

$3x + 5y + 7z = 12$ (1)

$x – 3y + 9z = 16$ (2)

$9x – 8y = 31z = 54$ (3)

Eliminate ‘*x*’ from equations (1) and (3)

$$\begin{array}{l} 9x + 15y + 21z = 36 \\ 9x + 8y + 31z = 54 \\ \underline{-\quad\ -\qquad -\qquad\ -\qquad} \\ 10z – 7y = 18 \end{array}$$ (5)

Equations (4) and (5) are same.

As equations 4 and 5 are same, it is not possible to find unique values of *x, y* and *z*.

45. $x_1 + 4x_2 + 9x_3 + 16x_4 + 25x_5 + 36x_6 = 9$ (1)

$4x_1 + 9x_2 + 16x_3 + 25x_4 + 36x_5 + 49x_6 = 98$ (2)

$9x_1 + 16x_2 + 25x_3 + 36x_4 + 49x_5 + 64x_6 = 987$ (3)

(2)–(1): $3x_1 + 5x_2 + 7x_3 + 9x_4 + 11x_5 + 13x_6 = 89$ (4)

(3)–(2): $5x_1 + 7x_2 + 9x_3 + 11x_4 + 13x_5 + 15x_6 = 889$ (5)

1. Total money = $3k + 2k + k = 120$

2. After round 2, *A* would have gained ₹ 20, which is the maximum gain for him in the course of the 4 rounds.

3. At ₹ 7 per km, the cost of travelling 25 km would be ₹ 175 and at ₹ 8 per km, the cost would be ₹ 200 (less than ₹ 240).

Hence, the taxi is hired for more than 4 hours, as the charge is ₹ 240.

Number of hours for which the taxi is hired = 240/48

= 5 hours.

4. Let the three digit number be *xyz*.

Given $3x - 2y = 3x - z = z - 2y$

$\Rightarrow 2y = z$ and $z = 3x$

$\Rightarrow x : y : z = 2 : 3 : 6$

$\therefore$ Only one number is possible under the given conditions, which is 236.

***Solutions for questions 5 and 6*:**

Let Sonali's age be *x* years

$\therefore$ Sagar's age is 2*x* years.

Let Monali's age be *y* years

$\therefore\ y + x = 2\ (2x) \Rightarrow y = 3x$

Let Surya's age be *z* years

$\therefore\ z + 2x = 2\ (x + 3x) \Rightarrow z = 6x$

Prithvi's age = 21 years

Now, $21 + 6x + 2x = 5(x + 3x) \Rightarrow 12x = 21 \Rightarrow x = 1^3/_4$

$\therefore$ Sonali's age = $1^3/_4$ years = 1 year 9 months

Sagar's age = $2\ (1^3/_4) = 3^1/_2$ years

Monali's age = $3\ (13/4) = 5^1/_4$ years

= 5 years and 3 months

Surya's age = $6\ (1^3/_4) = 10^1/_2$ years = 10 years 6 months

5. 5 years 3 months – 1 year 9 months = 3½ years

6. Let Surya be twice as old as sagar in '*t*' years.

$10^1/_2 + t = 2(3^1/_2 + t) \Rightarrow 10^1/_2 + t = 7 + 2t \Rightarrow t = 3^1/_2$

7. Let the amounts paid by Ram, Lakshman, Bharath and Shatrugna be *a*, *b*, *c* and *d*, respectively.

$\therefore$ a + *b* + *c* + *d* = ₹ 2,40,000 (1)

Given: $a = \frac{1}{2}(b + c + d)$

$\Rightarrow b + c + d = 2a$ (2)

From (1) and (2), *a* = ₹ 80,000

Now, $d = \frac{1}{5}(a + b + c)$

$\Rightarrow a + b + c = 5d$ (3)

From (1) and (3), *d* = ₹ 40,000

Also, $c = \frac{5}{19}\ (a + b + d)$

$\therefore\ a + b + d = \frac{19c}{5}$ (4)

From (1) and (4), *c* = ₹ 50,000

$\therefore$ *b* = 2, 40,000 – 80,000 – 50,000 – 40,000

= ₹ 70,000

8. Let the number of chocolates Raman had just before giving to the youngest son be *x*.

As he gave (2/3)*x* – 2 chocolates to the younger son, he would be left with (1/3) *x* + 2 chocolates (1/3) *x* + 2 = 18

$\Rightarrow x = 48$

Assume Raman had *y* chocolates before giving to the second son.

As he gave (1/4)y – 3 chocolates to the second younger, he would be left with 3/4*y* + 3

(3/4) *y* + 3 = 48, *y* = 60

Assume the number of chocolates Raman originally had was *z*.

As he gave (1/2)z – 4 chocolates to his eldest son, he would left with 1/2z + 4 chocolates

1/2z + 4 = 60

z = 112

***Solutions for questions 9 and 10*:**

Let the number of daughters in the Nanda family be *x* and that of sons be *y*.

$5(y - 1) = x$ (1)

and $x - 1 = 2y$ (2)

Solving (1) and (2), we get

$x = 5,\ y = 2$

Let the number of daughters in the Parekh family be *a* and that of the brothers be *b*.

$b - 1 = a$ (3)

$2(a - 1) = b$ (4)

Solving (3) and (4), we get; $a = 3,\ b = 4$

9. Parekh had 3 + 4 = 7 children.

10. Required ratio = (2 + 4) : (5 + 3) = 3 : 4

11. Let the present age of Ajay be x years

Some time in the past, Bharat was $x/4$ years old.

The age of Ajay at that time is Bharat's present age.

Bharat's present age is $(x - 9)$ years.

As their difference of ages is constant,

$x - (x - 9) = (x - 9) - x/4$

$x = 24$

Sum of their present ages = $2x - 9 = 39$ years

12. Let the number N be '*abc*'.

Let the sum of the remaining numbers be R.

$R + cba = R + abc + 11(a + b + c)$

$\Rightarrow 99(c - a) = 11(a + b + c)$

$\Rightarrow 8c - 10a = b$ (1)

As $b \geq 0$ and $c \geq \frac{5}{4}a$, $c >$ a

$\therefore$ 8 (Difference of a and c)

$= 8(c - a) = 6 + 2a$

$8c - 10a = 6$ (2)

From (1) and (2), $b = 6$

13. Let the number be '*abcd*'

Given that,

$b + c = 2d$ (1)

$b + 6a = 2(c + d)$ (2)

$d + 5a = 2b$ (3)

Let us that the equations (1), (2) and (3) as the linear equations in a, b and c and express the values of a, b and c in terms of b.

By (2) − (1), we get $3c = 6a \Rightarrow a = \frac{c}{2}$

By substituting $c = 2a$ in (1), it becomes

$2d - 2a = b$ (4)

Subtracting (4) from 2 × (3), we get

$12a = 3b \Rightarrow$ a $= \frac{b}{4}$

As $c = 2$a, $c = \frac{b}{2}$

By substituting $a = \frac{b}{4}$ in (4), we get $2d = \frac{3b}{2} \Rightarrow d = \frac{3b}{4}$

$\therefore a : b : c : d = 1 : 4 : 2 : 3$

$\therefore$ '*abcd*' can be 1423 or 2846.

14. Before doubling, the amounts with Bhavan, Chetan and Dinesh, each of them must have had $\frac{80}{2}$ = ₹40.

$\therefore$ Amar must have then had ₹80 + ₹120, i.e., ₹200. Similarly we can work out the amounts with each of them before the other doubled the amounts. The results are summarized below.

	Amar	Bhavan	Chetan	Dinesh
Finally	80	80	80	80
Before Amar doubles	200	40	40	40
Before Bhavan doubles	100	180	20	20
Before Chetan doubles	50	90	170	10
Before Dinesh doubles	25	45	85	165

Solutions for questions 15 and 16:

Let the number of erasers, pencils and pens be x, y and z, respectively.

$x + y + z = 38$ and $z > y > x$ and $x, y, z \geq 11$.

$\therefore x = 11$, and $y + z = 27$

There are two cases possible,

$\Rightarrow y = 12, z = 15$ or $y = 13, z = 14$

Let, $x = 12$, then $y + z = 26$

which is not possible because, if $y > x$, and $y = 13$, then $z = 13$ which would make $y = z$ which cannot be considered as $z > y$. Any other value of $y > 13$ would make $y > z$ which also is not acceptable.

$\therefore$ Two sets of values are possible.

$\therefore$ $x = 11, y = 12, z = 15$ and $x = 11, y = 13, z = 14$

15. Let the number of pens, pencils and erasers purchased be a, b and c respectively. It is given that $a > b > c \geq 11$. Also, it is known that he bought a total of 38 pieces.

$a + b + c = 38$

If $c = 11$, the following cases are possible:

$b = 12$ and $a = 15$

$b = 13$ and $a = 14$

If $c = 12$, the least value that b can take is 13 and the least value that a can take is 14, hence the sum will exceed 38. So, this case is ruled out.

Hence, $c = 11$.

16. If the number of pencils cannot be divided equally among the four brothers, then the number of pencils should not be a multiple of 12.

$\therefore$ The number of pencils = 13

$\therefore$ The number of pens = 14

17. We want the net score of Smitha to be 60 with maximum possible number of incorrect answers. Given that Smitha

From Statement II, the number of windows and the number of doors are in the ratio of 2 : 1. This information alone is not sufficient to answer the question. Using I and II, $36 - x = 2x \Rightarrow x = 12$.

14. As Statements I and II individually do not relate to Uno and Zen, they are not sufficient.

Using both, we have Palio = 4/5 (Alto);

Alto = Zen – 2

Also, Alto gives $45 \times \frac{4}{3} = 60$ km

∴ Zen = 62 km

∴ Zen gives more mileage.

15. From Statement I, we have the price of gold per ten grams = ₹ 4000.

Price of silver per 1 kg = ₹ 8000

∴ The price of silver per 10 grams = ₹ 8

∴ The price of gold per unit weight = 500 times of price of silver per unit weight

∴ Statement I alone is sufficient

Statement II does not give any relevant information.

16. Two equations $a_1x + b_1y + c_1z = k_1$ and $a_2x + b_2y + c_2z = k_2$

have a unique value for z only if $\frac{a_1}{a_2} = \frac{b_1}{b_2}\left(\neq \frac{C_1}{C_2}\right)$.

Combining the given equations with the equation in either Statement, the above condition is not satisfied. Combining both Statements, as we have three equations and three unknowns. A unique solution in z is possible.

Hence, both Statements taken together are sufficient.

17. Using Statement I, multiplying the equation in I by 5 and subtracting the given equation from it, the value of $7x + 16y - 2z$ is obtained. $7x + y - 2z = 7x + 16y - 2z - 15y$. As y is unknown, $7x + y - 2z$ cannot be found.

∴ Statement I alone is not sufficient.

Using Statement II, $3(3x - y + 2z) - 2(x - 2y + 4z)$

$= 7x + y - 2z$

∴ $7x + y - 2z = 3(11) - 2(12) = 9$

∴ Statement II alone is sufficient.

18. After the coins are exchanged the possible combinations of notes that Guru has are as follows.

₹ 50	₹ 100	Total
2	4	6
4	3	7
6	2	8
8	1	9

From Statement I, It follows that he has two 50 rupee notes.

∴ Statement I alone is sufficient

From Statement II, it follows that he has four, six or eight 50 rupee notes.

∴ Statement II alone is not sufficient.

19. Let my current age be x years and my sister's current age be y years.

Using either statement, we get one equation in two unknowns.

∴ x cannot be found.

∴ Either statement alone is not sufficient.

Using both statements, as we have two equations in two unknowns, x and y can be found

20. Using Statement I, $12x + 18y = 18a \Rightarrow 2x + 3y = 3a$

∴ It has integral solutions (Eg: $x = 0$, $y = a$)

∴ Statement I alone is sufficient.

Using Statement II,

$c^2 + 4c - 396 = 0$

$(c - 18)\ (c + 22) = 0$

If $c = 18$, there are integral solutions

If $c = -22$, there are no integral solutions

∴ Statement II is not sufficient.

21. Dividing both numerator and denominator of $\frac{7a + 9b}{4a + 5b}$ by b, it becomes $\frac{7\frac{a}{b} + 9}{4\frac{a}{b} + 5}$. Value of $\frac{a}{b}$ is sufficient to find $\frac{7a + 9b}{4a + 5b}$.

From Statement I, we have $b + a = \frac{1}{2}(6a - b)$

$\Rightarrow {}^{a}/_{b} = {}^{3}/_{4}$. The value of the expression can be found.

∴ Statement I is sufficient.

From Statement II, we have $3a + 4b = 5$.

By simplifying $3a + 4b = 5$, $\frac{a}{b}$ cannot be found.

∴ Statement II is not sufficient.

22. Using either of the Statements, we get one equation in two unknowns. Therefore, A's speed cannot be found. Combining both Statements, as we have two equations in two unknowns A can be found.

23. Let the cost of an apple and a mango be ₹ x and ₹ y, respectively.

$3x + y = 14$ (1)

Using Statement I, $5x + 2y + 13 = 6x + 4y$

$\Rightarrow x + 2y = 13$ (2)

Solving (1) and (2), $y = 5$.

Using Statement II, $x = 2y$

Substituting $x = 2y$ in (1),

$y = 2$.

∴ Statement II alone is sufficient.

24. Two equations $a_1\ x + b_1\ y = c_1$ and $a_2\ x + b_2\ y = c_2$ have a unique solution if and only if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. Combining the given equations with equation in Statement I, the above condition is not satisfied. Combining it with equation in Statement II, this condition is satisfied.

∴ Statement II alone is sufficient.

25. Let y people read both.

Using Statement I, 1500 read The Hindu and 900 read The Times of India. We can't determine the number of people who read only The Hindu.

∴ Statement I is not sufficient.

Statement II is clearly not sufficient as nothing is said about any newspaper.

Combining both Statements, $1500 + 900 - y = 2000y = 400$

26. From Statement I,

If Amisha purchases equal number of apples and oranges, (say n of each) she must pay ₹ $12n$

Since $65 \neq 12n$, she has not bought equal number of apples and oranges. Hence, I alone is sufficient.

From Statement II alone, had she bought 3 oranges and 4 apples more, she would have paid ₹ 108.

So, she actually paid $108 - 3 \times 5 - 4 \times 7 = 65$.

This is now same as Statement I and hence, II alone is sufficient.

27. Neither of the Statements is independently sufficient, as we have three unknowns and two equations from the two Statements. Using both the Statements, we have the following equations.

$5P + 7E + 15S = 49$ (1)

$8P + 11E + 23S = 77$ (2)

By multiplying the first equation by 3 and subtracting the second equation multiplied by 2 from the resulting equation, will give us the cost of $1P + 1S + 1E$.

28. Let the four-digit number be abcd.

Using Statement I, $a + b + c = d = 3a$

Using Statement II, $b = 2a$ and $c = a - 2$

Using both the Statements,

$a + b + c = a + 2a + (a - 2)$

$4a - 2 = 3a$ as $a + b + c = 3a$

$\Rightarrow a = 2, b = 4$ and $c = 0$

$d = a + b + c = 6$

Hence, the number is 2406

29. Let the present age of Amit be a years

Let the present age of Bimal be b years

Given

$a + x = 2(b + x)$ (1)

Also,

$b + 2x = a$ (2)

From (1) and (2), we have

$b + 3x = 2b + 2x$

$b = x$

$x + 2x = a$ so $a = 3x$

From Statement I

$a - b = 5$

Thus, $3x - x = 5$

Hence, x can be found.

So, Statement I alone is sufficient.

From Statement II, $a + 2x = 5b$

As $a = 3x$ and $b = x$, we get $5x = 5x$.

From this, x can't be found out.

Statement II alone is not sufficient.

30. Let the amounts with Ram and Shyam be ₹ R and ₹ S, respectively. Given that $R + S = 200$

If Ram has more than ₹ 100, he would have more than Shyam. If he does not have more than ₹ 100, he would not have more than Shyam.

Using Statement I, if Ram gives ₹10 to Shyam, Ram and Shyam would have ₹ $(R-10)$ and ₹ $(S+10)$, respectively.

There are two possibilities.

(1) $R-10>S+10$. In this case,

$R-10-(S+10)=20$, i.e.,

$R-S=40$

(2) $R-10<S+10$. In this case, $S+10-(R-10)$ $=20$, i.e., $S=R$

$\therefore$ $R-S=40$ or $R=S$.

In the first case, Ram has more than Shyam. In the second case, he does not have more than Shyam. I is not sufficient.

Using Statement II, $S-5-(R+5)=10$ or $R+5-(S-5)$ $=10$, i.e., $S-R=20$ or $S=R$. In both cases, Ram does not have more money than Shyam. II is sufficient.